9/06/2006

$\{a, a, b, b, c, d, d\} \neq \{a, b, c, d\}$

multi-set

$\{2 \times a, 2 \times b, 1 \times c, 2 \times d\}$

For a set of size n,

there are $n!$ permutations

Size of multi-set = # elements

Size 7

How many permutations?

$\{2\times a, 2\times b\}$

aabb abab abba

bbaa baba baab

<u>only 6</u>

$\{a_1, a_2, b_1, b_2\}$

has $4! = 24$ permutations

Write these out

$a_1 a_2 b_1 b_2$

$a_1 a_2 b_2 b_1$ then erase subscripts

⋮

aabb appears 4 times

$a_1 a_2 b_1 b_2$ | 4

$a_1 a_2 b_2 b_1$ | 4 | 24

$a_2 a_1 b_1 b_2$ | 4

$a_2 a_1 b_2 b_1$

⋮

6 blocks | 4

$\equiv$ 6 permutations of the multi-set

In general

$$\{a_1 \times 1, a_2 \times 2, \ldots, a_n \times n\}$$

$$a_1 + a_2 + \ldots + a_n = m$$

perms =

$$\frac{m!}{a_1!\, a_2! \cdots a_n!}$$

multi-nomial coefficient

$$\binom{m}{a_1, a_2, \ldots a_n}$$

$$(x_1 + x_2 + \cdots + x_n)^m =$$

$$\sum_{a_1 + a_2 + \cdots = m} \binom{m}{a_1, \ldots, a_n} x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n}$$

$(x_1 + \dots + x_n) \times (x_1 + \dots + x_n) \times \dots$

How do we get terms

$$x_1^3 \, x_2^6 \, x_3^4 \dots$$

Pick 3 x_1's
$+$ 6 x_2's
$+$ 4 x_3's
$+$

$\overline{}$
m

1 1 1 2 2 2 2 2 2 2 2 3 3 3 3 or any perm.

Balls in boxes

[For CS people: HASHING]

m distinguishable balls

$\Downarrow$

n boxes

Box i gets b_i balls.

$\#\text{ways} = \binom{m}{b_1, b_2, \ldots, b_n}$.

Inclusion-Exclusion

$$|\bar{A}_1 \cap \bar{A}_2|$$

$$= |A| - |A_1| - |A_2|$$

$$+ |A_1 \cap A_2|$$

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

$$|A| - |\bar{A}_1 \cap \bar{A}_2|$$

$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3 \cap \bar{A}_4|$ $N=4$

$= |A|$

$-|A_1| - |A_2| - |A_3| - |A_4|$

$+|A_1 \cap A_2| + |A_1 \cap A_3| + \dots + |A_3 \cap A_4|$

$-|A_1 \cap A_2 \cap A_3| - \dots - |A_2 \cap A_3 \cap A_4|$

$+ |A_1 \cap A_2 \cap A_3 \cap A_4)$

$$\left|\bigcap_{i=1}^{N} \overline{A}_i\right|$$

$$= \sum_{S\subseteq[N]} (-1)^{|S|} |A_S|$$

$$A_S = \bigcap_{i\in S} A_i$$

How many integers in [10,000] are not divisible by 4, 6, 7

$A = \{1, 2, \ldots, 10{,}000\}$

$A_1 = \{4, 8, \ldots, 10{,}000\}$

$A_2 = \{6, 12, \ldots, 9996\}$

$A_3 = \{7, 14, \ldots 9996\}$

$?? = |\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3|$??

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| =$$

$$10000 - \underset{|A_1|}{2500} - \underset{|A_2|}{1666} - \underset{|A_3|}{1428}$$

$$+ \underset{|A_1 \cap A_2|}{833} + \underset{|A_1 \cap A_2|}{357} + \underset{|A_2 \cap A_3|}{238}$$

$$- \underset{|A_1 \cap A_2 \cap A_3|}{119}$$

$$=$$