9/1/2006

$$\frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,(n-(n-r))!}$$

Pascal's $\triangle$

Base Case

$$\binom{k}{k} = \binom{k+1}{k+1}$$

Choose a set of size

$k+1$.

Look at largest element

$S_m = \{$ sets for which $\downarrow$ is $m+1$ $\}$

$m = k, k+1, \ldots, n$.

S_m are a partition

$$|S| = |S_k| + |S_{k+1}| + \dots + |S_n|$$

$|S_m| = \#$ $(k+1)$ - subsets

which $m+1$ and only

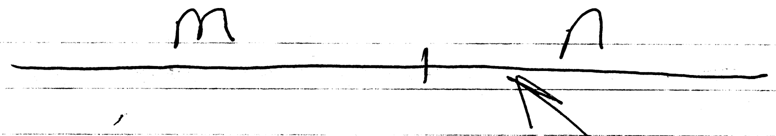
Smaller elements

k from here $\qquad$ $m+1$

$\#\text{choices} = \binom{m}{k}$

Vandermonde Identity

$$\underbrace{\sum_{r=0}^{k} \binom{m}{r}\binom{n}{k-r}}_{\text{What is this?}} = \binom{m+n}{k}$$



choose k from

Fix r: choose r from left #ways $= \binom{m}{r}$

$k-r$ from right $\binom{n}{k-r}$

#choices is $\binom{m}{r} \times \binom{n}{k-r}$

Binomial Theorem

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

Coeff: of x^r

$(1+x)(1+x) \cdots (1+x)$

Put $x=1$

$$2^n = \sum_{r=0}^{n} \binom{n}{r} \leftarrow \text{Total \# subsets of } [n]$$

$x = -1$

$$(1-1)^n = \binom{n}{0} + \binom{n}{1}(-1) + \binom{n}{2}(-1)^2 + \cdots$$

$$= \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots$$

$$0 = \left[\binom{n}{0} + \binom{n}{2} \cdots\right]$$

$$- \left[\binom{n}{1} + \binom{n}{3} + \cdots\right]$$

$$(1+i)^n = \sum_{k=0}^{n} \binom{n}{k} i^k$$

LHS: $\frac{1+i}{\sqrt{2}} = \cos\frac{\pi}{4} + i \sin\frac{\pi}{4}$

$= e^{i\pi/4}$ $\quad \cos\theta + i\sin\theta = e^{i\theta}$

$(1+i)^n = 2^{n/2} e^{in\pi/4}$

$= \boxed{2^{n/2} \cos\frac{n\pi}{4}} + i\, 2^{n/2} \sin\frac{n\pi}{4}$

RHS: Real Bit: $\boxed{\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \cdots}$

Complex Bit $\binom{n}{1} - \binom{n}{3} + \cdots - - -$

$\binom{n}{3} i^3$

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

Differentiate

$$n(1+x)^{n-1} = \sum_{k=0}^{n} k\binom{n}{k} x^{k-1}$$

Put $x=1$.

$$n2^{n-1} = \sum_{k=0}^{n} k\binom{n}{k}$$

Why?

Both sides count something?

# of pairs (x, S) where $x \in S$

chair committee