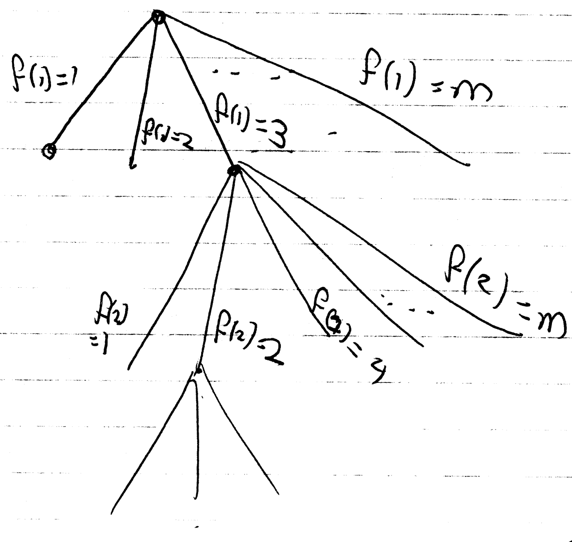
8/30/2006

$\phi_{1-1}(m,n) = \#_{1-1}$ mappings $[n] \longrightarrow [m]$

Claim: $\phi_{1-1} = m(m-1)\cdots(m-n+1)$

Nb: $m > n \Rightarrow \phi_{1-1} = 0$.



vertices in tree

ϕ_{1-1} counts some sequence

$a_1\, a_2 \cdots a_i \cdots a_n$

$\uparrow$

$a_i \in [m] \quad a_i = f(i)$

$1\text{-}1 \implies a_i \neq a_j \quad i \neq j$

$m = n$

We get permutations of $[n]$.

$$\binom{n}{k} \equiv n \text{ choose } k$$

$$= \frac{n!}{k!\,(n-k)!}$$

$$= \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 1}$$

$\binom{X}{k}$ X is a set.

— Collection of k-subsets of X.

$$\underbrace{k!\left|\binom{X}{k}\right|} \longleftarrow \#\text{ of ways}$$

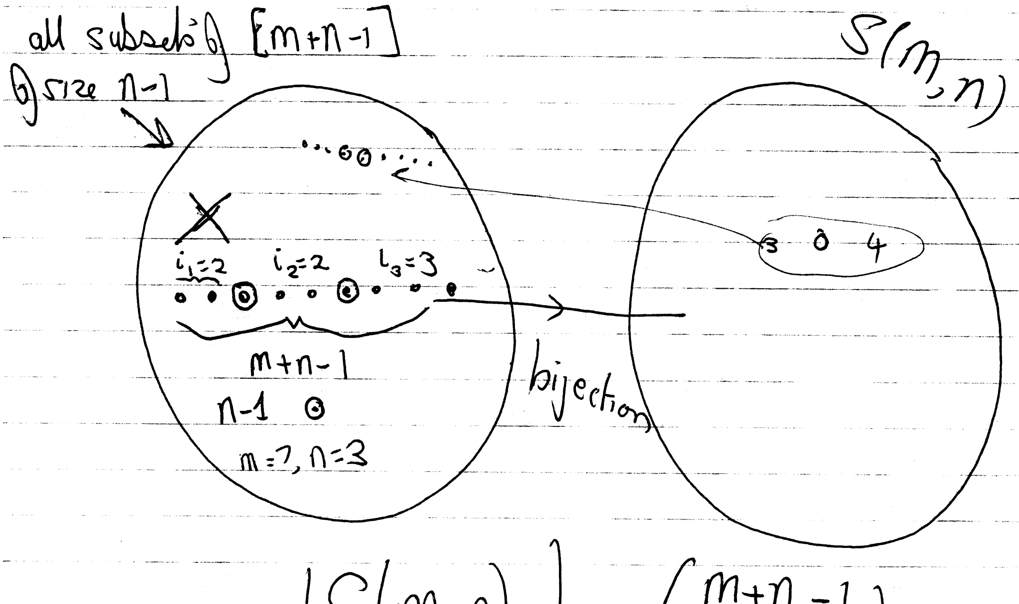
Take a k-subset of X

and then order it.

~~x~~ n

$$S(m,n) = \{ i_1, i_2, \ldots, i_n \in \mathbb{Z}^+ : i_1 + i_2 + \ldots + i_n = m \}$$

$$|S(m,n)| = ?$$



$$|S(m,n)| = \binom{m+n-1}{n-1}$$

Colouring ~~in~~ indistinguishable balls

m balls $m = 4$

n colours $n = 3$

(i) If balls are distinguishable : ~~m^{n}~~ n^{m}

(ii) Indistinguishable

RRRR RRRB BBBR GGGR
BBBB RRRG BBBG GGGB
GGGG

RRBB RRGG BBGG

3

RRBG BBRG GGRB

$$\binom{4+3-1}{3-1} = \binom{6}{2} = 15$$

ways of colouring m indistinguishable balls with n colours is $\binom{m+n-1}{n-1}$

i_t = # balls that colour t.

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ways where each colour uses ≥ 1 time. $N = \{1, 2, 3, \ldots\}$