21-301 Combinatorics
Homework 8
Due: Monday, November 7

1. Let $s \geq 1$ be fixed. Let $\mathcal{A} \subseteq \mathcal{P}_n$ be such that **there do not exist** distinct $A_1, A_2, \ldots, A_{s+1}$ such that $A_1 \subseteq A_2 \subseteq \cdots \subseteq A_{s+1}$. Show that
$$\sum_{A \in \mathcal{A}} \frac{1}{\binom{n}{|A|}} \leq s.$$

2. Show that if $n \geq 1$ and $S \subseteq [2n]$ and $|S| \geq n+1$ then there exist distinct $a, b \in S$ such that a divides b.

3. Let $m = n^4 + 1$. Given two sequences $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$ of real numbers, show that there exist $i_1, i_2, \ldots, i_{n+1}$ such that *both* subsequences $a_{i_1}, a_{i_2}, \ldots, a_{i_{n+1}}$ and $b_{i_1}, b_{i_2}, \ldots, b_{i_{n+1}}$ are monotone.

   Here we do not insist that both are monotone increasing or both are monotone decreasing, so one can be increasing and the other decreasing.