21-301 Combinatorics
Homework 7
Due: Monday, October 31

1. Show without using the Cayley formula that if T_n denotes the number of trees on n given vertices then

$$(n-1)T_n = \sum_{k=1}^{n-1} k(n-k)\binom{n-1}{k-1} T_k T_{n-k}.$$

2. Show that there are $2^{\binom{n-1}{2}}$ graphs on vertex set $[n]$ which do not have any vertices of odd degree.

3. Every pair of odd cycles of the graph G intersect in at least one vertex. Show that G has chromatic number at most 5.
(There is a very simple proof of this. Don't try anything complicated.)