21-301 Combinatorics Homework 5 Due: Monday, October 10

- 1. A box has three drawers; one contains two gold coins, one contains two silver coins and one contains one gold and one silver coin. Assume that one drawer is selected randomly and that a randomly selected coin from that drawer turns out to be gold. What is the probability that the chosen drawer is the one with two gold coins?
- 2. A bag contains n balls, each of a different color. In a round, a person picks a random ball from the bag, makes a note of its color and then puts it back. What is the expected number of rounds required for the person to have pulled out a ball of each color at least once?
- 3. A particle sits at the left hand end of a line $0 1 2 \cdots L$. When at 0 it moves to 1. When at $i \in [1, L - 1]$ it makes a random move to i - 1 or i + 1 with equal probability. When at L it stops. Show that the expected number of visits to 0 during the whole random walk is L. Here the count of visits starts at 1 with the particle being at 0.

(Hint: Let E_k denote the expected number of visits if we started the walk at k. Construct a set of equations satisfied by the E_k 's. What we want is E_0 .)