21-301 Combinatorics
Homework 3
Due: Monday, September 19

1. A group of $2n$ people is seated around a round table. The group leaves the table for a break and then returns. In how many ways can the people sit down after returning so that no one is sitting directly opposite to the same person in both seatings? (If the positions round the table are $1, 2, \ldots, 2n$ then $n+i$ is directly opposite i for $1 \le i \le n$).

2. Show that the number of permutations of $[n]$ which do not contain a consecutive pair of the form $k, k + 1$ satisfies the recurrence

$$b_n = (n - 1)b_{n-1} + (n - 2)b_{n-2}.$$

   [Hint: delete n from such a sequence and separately count those permutations which still satisfy the condition and those that don't.]

3. Let $a_0, a_1, a_2, \ldots$ be the sequence defined by the recurrence relation

$$a_n = a_{n-1} + 2a_{n-2} + 1 \quad \text{for } n \ge 2$$

   with initial conditions $a_0 = 0$ and $a_1 = 1$. Determine the generating function for this sequence, and use the generating function to determine a_n for all n.