21-301 Combinatorics
Homework 1
Due: Wednesday, September 14

1. Use the binomial theorem to prove that

$$\binom{n}{0} - \binom{n}{2} + \binom{n}{4} - \binom{n}{6} + \cdots = 2^{n/2} \cos n\pi/4.$$

$$\binom{n}{1} - \binom{n}{3} + \binom{n}{5} - \binom{n}{7} + \cdots = 2^{n/2} \sin n\pi/4.$$

[Hint: $\cos \theta + i \sin \theta = e^{i\theta}$ where i denotes $\sqrt{-1}$. Now consider the expansion of $(1+i)^n$.]

2. Show that for a fixed k,

$$\sum_{\ell=0}^{n-k} \binom{n}{k, \ell, n-k-\ell} = 2^{n-k} \binom{n}{k}.$$

[Hint: Expand $(1+x+y)^n$ using the multinomial theorem. Then put $y = 1$ and extract the coefficient of x^k in what remains.]

3. Of the 16! orderings of the letters $A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P$ how many are there such that we cannot obtain any of the words BAD, DEAF, APE by crossing out some letters?