21-301 Combinatorics Homework 10 Due: Monday, December 5

- 1. Consider the following game: There is a pile of n chips. A move consists of removing any *proper* factor of n chips from the pile. The player to leave a pile with one chip wins. Determine the N and P positions and a winning strategy from an N position.
- 2. Two players play the following game on a round table top of radius R. Players take turns placing pennies (of unit radius) on the tabletop, but no penny is allowed to overlap. Which of the players has a winning strategy as a function of R and why?
- 3. Consider the following game: There is a single pile of n chips. A move consists of removing (i) any number of chips divisible by three provided it is not the whole pile, or (ii) the whole pile, but only if it has 2 (mod 3) chips. The terminal positions are zero, one and three. Determine the Sprague-Grundy numbers of each pile size.