

21-301 Combinatorics
Homework 1
Due: Wednesday, September 7

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -5$ and $x_4 \geq 8$?

2. Prove the following equality using a *combinatorial* argument

$$\sum_{i=1}^n i \binom{n}{i} = n2^{n-1}.$$

3. Show that the number of functions $f : [n] \rightarrow [n]$ which satisfy $i < j$ implies $f(i) \leq f(j)$ is $\binom{2n-1}{n}$.
(Hint: Consider the jumps $f(i+1) - f(i)$.)