Department of Mathematics
Carnegie Mellon University

21-301 Combinatorics, Fall 2005: Test 3

Name:_____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Let n, p be positive integers and let $N = n^2p+1$. Suppose that $x_1, x_2, \ldots, x_N$ are real numbers. Show that either (i) there is a *strictly* increasing subsequence of length $n+1$ or (ii) a *strictly* decreasing subsequence of length $n+1$ or (iii) a *constant* subsequence of length $p+1$.

Q2: (33pts)
A tree T has $n = 2m$ vertices. All vertices of T have degree one or three. There are n_1 vertices of degree one and n_3 vertices of degree three. Determine the values of n_1, n_3 in terms of m. Justify your claim.
How many such trees are there on vertex set $\{1, 2, \ldots, n\}$?

Q3: (34pts)
Let p, q be positive integers and $n = p + q - 1$. Let T be a fixed tree with q vertices. Show that if we color the edges of K_n Red or Blue then either (i) there is a vertex with Red degree p or (ii) there is a Blue copy of T.