Department of Mathematics
Carnegie Mellon University

21-301 Combinatorics, Fall 2005: Test 2

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) A box has four drawers; one contains three gold coins, one contains two gold coins and a silver coin, one contains a gold coin and two silver coins and one contains three silver coins. Assume that one drawer is selected randomly and that a randomly selected coin from that drawer turns out to be gold. What is the probability that the chosen drawer is the one with three gold coins?

Q2: (33pts) Let $A_1, A_2, \ldots, A_n$ be subsets of A with $|A_i| = k$ for $1 \leq i \leq n$. Show that if $n(2^{1-k} + k2^{2-k}) < 1$ then it is possible to partition the set A into two sets R, B (i.e. color A red and blue) so that

$$|A_i \cap R| \geq 2 \text{ and } |A_i \cap B| \geq 2 \text{ for } i = 1, 2, \ldots, n.$$

Q3: (34pts)

A particle sits at the left hand end of a line $0 - 1 - 2 - \cdots - L$. When at 0 it moves to 1. When at $i \in [1, L-1]$ it makes a move to $i-1$ with probability 1/3 and a move to $i+1$ with probability 2/3. When at L it stops.

Let E_k denote the expected number of visits to 0 if we started the walk at k.

1. Find a set of equations satisfied by the E_k.

2. Given that $E_k = \frac{A}{2^k} + B$ is a solution to your equations for some A, B, determine A, B and hence find E_0.