Department of Mathematics
Carnegie Mellon University

21-301 Combinatorics, Fall 2005: Test 1

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) Let a, m, n, p be positive integers. How many integer solutions are there to

$$x_1 + x_2 + \cdots + x_m = n$$

which satisfy $x_i \geq a$ for $i = 1, 2, \ldots, p$ and $x_i \geq 0$ for $i = p+1, p+2, \ldots, m$.

Q2: (33pts) Use the inclusion-exclusion formula

$$\left|\bigcap_{i=1}^{N} \overline{A}_i\right| = \sum_{S \subseteq [N]} (-1)^{|S|} |A_S|$$

to show that the number of permutations $\pi(1), \pi(2), \ldots, \pi(n)$ of $[n]$ which satisfy $\pi(i+1) \neq \pi(i) + 1$ for $i = 1, 2, \ldots, n-1$ is

$$\sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} (n-k)!.$$

Q3: (34pts) The sequence $a_0, a_1, \ldots, a_n, \ldots$ satifies the following:
$a_0 = 1$, $a_1 = 4$ and

$$a_n - 4a_{n-1} + 4a_{n-2} = 0$$

for $n \geq 2$.
Determine the generating function $a(x) = \sum_{n=0}^{\infty} a_n x^n$ and hence find a_n.