Combinatorial Analysis 21-301: Fall 2003
Homework.
HW8 due Monday 11/3/2003

Q1: A necklace is made of 6 beads strung together in a cycle. Compute the pattern inventory for colouring the beads using two colours if the group G is the **dihedral group** generated by rotations about the centre plus flips about a diameter. Using the notation R to represent a clockwise rotation of $\pi/3$ and F to represent a flip on the diameter 1–4, the group G consists of the 12 elements $\{R^i F^j : \ 0 \leq i \leq 5,\ j = 1, 2\}$.
Here we can reduce any sequence of rotations and flips to the 12 given by using the relations $R^6 = F^2 = e$ (identity) and $FR = R^5 F$.
See http://merganser.math.gvsu.edu/david/reed03/projects/ettingerGuy/ for a nice discussion.

Q2: Consider a complete binary tree on 7 nodes (drawn below): Compute the pattern inventory for colouring the edges using two colours if the group G can rotate the tree below each vertex. This group has 8 elements: Let e_a, e_b, e_c denote rotations under vertices a, b, c respectively. Then dropping the $\circ$ we have $e_a e_b = e_c e_a$, $e_a e_c = e_b e_a$ and $e_b e_c = e_c e_b$. So G consists of $\{e, e_a, e_b, e_c, e_a e_b, e_a e_c, e_b e_c, e_a e_b e_c\}$.