Combinatorial Analysis 21-301: Fall 2003
Homework.
HW5 due Monday 10/6/2003

Q1: Let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ be a collection of m distinct subsets of $[n]$ such that if $i \neq j$ then $A_i \cap A_j \neq \emptyset$. Show that $m \leq 2^{n-1}$ and give an example where $m = 2^{n-1}$.

Q2: Let $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ be a collection of m distinct subsets of $[n]$ such that if $i \neq j$ then (i) $A_i \not\subseteq A_j$, (ii) $A_i \cap A_j \neq \emptyset$ and (iii) $A_i \cup A_j \neq [n]$. Prove that

$$m \leq \binom{n-1}{\lfloor n/2 \rfloor}.$$

[Hint: Show that replacing sets of size greater than $n/2$ leaves a set system in which (i) and (ii) hold. Then apply Theorem 6.6 (Bollobás).]