Combinatorial Analysis 21-301: Fall 2003 Homework. HW4 due Monday 9/29/2003

Q1: Show that a simple graph G with n vertices and m edges has at least $\frac{m}{3n}(4m - n^2)$ triangles.

[Hint: An edge (x, y) is in at least deg(x) + deg(y) - n triangles. Sum this over all edges of G.]

Q2: Use König's Theorem to prove that every bipartite graph G has a matching of size at least $|E(G)|/\Delta(G)$ where Δ denotes the maximum degree. Use this to show that every subgraph of $K_{n,n}$ with more than (k-1)n edges has a matching of size at least k.