Combinatorial Analysis 21-301: Fall 2003
Homework.
HW3 due Monday 9/15/2003

Q1: The edges of $K_n, n \geq 4$ are coloured Red and Blue in such a way that a Red edge is in at most one triangle. Show that there is a subgraph K_k with $k \geq \lfloor \sqrt{2n} \rfloor$ that contains **no** Red triangles.
[Hint: Choose a maximal set K of k vertices that do not contain a Red triangle. Now use the fact that every vertex not in K makes a Red triangle with an edge contained in K. Also use the fact (to be shown) that if K contains no Red triangles then K must contain at least $k/2$ Blue edges.]

Q2: Let m be given. Show that if n is large enough then every $n \times n$ 0-1 matrix contains a **principal** $m \times m$ sub-matrix in which all elements below the diagonal are the same, and all elements above the diagonal are the same. (A principal submatrix is one made up of elements from rows $i_1, i_2, \ldots, i_m$ and columns $i_1, i_2, \ldots, i_m$, for some $i_1, i_2, \ldots, i_m$.
[Hint: If A is an $n \times n$ 0-1 matrix, colour K_n with *colour* $A_{i,j}A_{j,i}$ for $i < j$ i.e. with 00 or 01 or 10 or 11.]