Combinatorial Analysis 21-301: Fall 2003
Homework.
HW2 due Monday 9/8/2003

Q1: A *graceful labelling* of a tree T on n vertices is a mapping from $V(T) \to [n]$ so that the numbers $|f(x) - f(y)|$, computed across edges, are all different. Show that a path has a graceful labelling. (It is conjectured that all trees have graceful labellings, but you are not expected to settle this conjecture).

Solution Label the vertices $n, 1, n-1, 2, n-2, \ldots, \lceil n/2 \rceil$.

Q2: A tree T has exactly one vertex of degree i for each $2 \leq i \leq m$ and all other vertices are of degree one. How many vertices does T have? Justify your answer, (of course).

Solution Let n be the number of vertices in T and k be the number of vertices of degree 1. Then

$$\begin{aligned} n &= k + m - 1 \\ 2n - 2 &= k + 2 + \cdots + m = k + \frac{m(m+1)}{2} - 1 \end{aligned}$$

The second equation says that 2 times number of edges = sum of degrees in T.

Solving, we obtain

$$n = \frac{m^2 - m + 4}{2}.$$