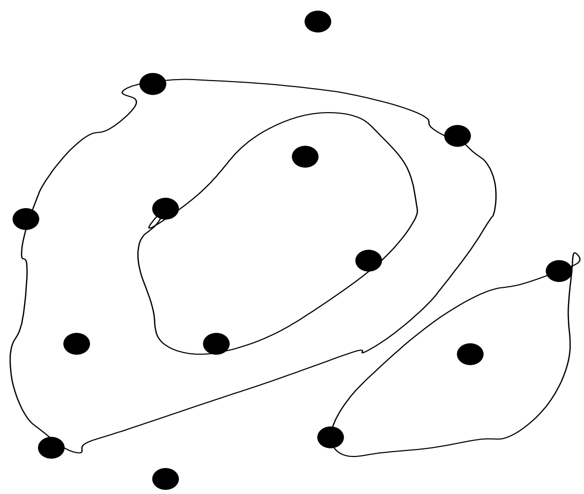
Combinatorial Analysis 21-301: Fall 2003
Homework.
HW10 due Monday 11/24/2003

Q1: In **Empty and Divide** there are two boxes. Initially, one box contains m chips and the other contains n chips. Such a position is denoted by (m, n) where $m > 0$ and $n > 0$. A move consists of emptying one of the boxes and dividing the contents of the other between the two boxes with at least one chip in each box. There is a unique terminal position, namely (1,1). The last player to move wins. Determine which positions are P-positions and which positions are N-positions and how to win the game from an N-position.
Solution (m, n) is a P-position iff both of m and n are odd. To see this observe that the unique sink $(1, 1)$ satisfies this condition. Then if both m and n are odd and $(m, n) \neq (1, 1)$, then the next move is to (m', n') where $m' + n'$ is odd. So one of m', n' is even. Finally, if m say, is even then we can move to (m', n') where $m' + n' = m$ and both m', n' are odd.

Q2: Analyse the following variant of Nim and then show that Rims below is this game in disguise. After removing chips from a pile, a player can if so desired, split the remainder of the pile into two sets. The winner is still the player that takes the last chip.
Rims A position in the game of Rims is a finite set of dots in the plane, possibly separated by non-intersecting closed curves. A move consists of drawing a closed curve through any positive number of dots but not touching any other curve. Players alternate moves and the last to move wins.



Solution If the number of chips in each pile is $x = (x_1, x_2, \ldots, x_n)$ then we can still take $g(x) = x_1 \oplus x_2 \oplus \cdots \oplus x_n$. This is because if $g(x) = 0$ and a move creates x' then $g(x') \neq 0$. Also, if $g(x) \neq 0$ then there is a regular Nim move to x' with $g(x') = 0$.
Finally, imagine cutting around the curves and letting the plane fall to pieces. The dots on a piece represent a pile and drawing a curve through points is equivalent to deleting a set of chips. The inside and outside of the curve represent the partition into two sub-piles.