Combinatorial Analysis 21-301: Fall 2003
Homework.
HW1 due Monday 9/1/2003

Q1: Let $A_1, A_2, \ldots, A_n$ be n distinct subsets of $[n]$. Show that there is an element $x \in [n]$ such that all of the sets $A_i \setminus \{x\}$ are also distinct.

Proof method: Consider the graph G with **vertices** $A_1, A_2, \ldots, A_n$ and an edge $\{A_i, A_j\}$ with "colour" x whenever $A_i \oplus A_j = \{x\}$. Prove that in any cycle of G, a colour can appear an even number of times. Deduce that one can delete edges of G so that no cycles are left and the number of colours remains the same. Then use the fact that a graph with n vertices and no cycles contains at most $n-1$ edges.

Q2: Let G be a simple graph with $n > 3$ vertices and no vertex of degree $n-1$. Suppose that for any two vertices of G there is a **unique** vertex joined to both of them. Prove

(i) If x and y are non-adjacent then they have the same degree.

(ii) Now show that G is a **regular** graph i.e. every vertex has the same degree.

[Hint for (i): Suppose that the x, y are non-adjacent. Let the neigbours of x be $x_1, x_2, \ldots, x_k$ and the neighbours of y be $y_1, y_2, \ldots, y_l$, with $x_1 = y_1$. Show that each y_i is adjacent to a unique member of $\{x_1, x_2, \ldots, x_k\}$.]