QUANTUM PHYSICS AND MATCHINGS IN GRAPHS

This project is based around the concepts and the questions discussed in the paper [KGS19] by Krenn, Gu, and Soltész. It has been recently discovered that certain problems in quantum physics can be phrased in purely combinatorial terms as questions about matchings in graphs. The goal of this project is to explore the resulting graph-theoretic notions.

The main object of study here is an edge bi-colored weighted graph, or simply a bi-colored graph. It is defined as follows. Fix a finite set $C$ of $k \geq 2$ colors. Let $G$ be a finite graph. To each edge $e \in E(G)$, a complex weight $\omega_e$ is assigned. Each edge $e = uv$ also receives an (ordered) pair of colors from $C$: $c_e(u)$ and $c_e(v)$ (that is, the two colors are indexed by the endpoints of $e$). We say that $c_e(u)$ is the color of $e$ at $u$, while $c_e(v)$ is the color of $e$ at $v$.

Recall that a perfect matching in a graph $G$ is a subset $M \subseteq E(G)$ of edges such that each vertex of $G$ is incident to precisely one edge in $M$. If $G$ is a bi-colored graph, then each perfect matching $M$ in $G$ gives rise to a vertex coloring $c_M$ of $G$, defined as follows. Take any $u \in V(G)$. Then $u$ is incident to precisely one edge $e \in M$, and we set $c_M(u) := c_e(u)$.

If $c$ is a vertex coloring of a bi-colored graph $G$, then the weight of $c$ is defined as

$$\omega(c) := \sum_M \prod_{e \in M} \omega_e,$$

where the sum is taken over all the perfect matchings $M$ with $c_M = c$. This definition captures some kind of interference phenomena in quantum physics. In particular, if $\omega(c) = 0$, then we say that the coloring $c$ cancels out. Since the importance of these notions to quantum physics has only recently been brought to light, very little is known about them. For instance, the following question is open:

**Question 1.** For which values of $n$ and $k$ there is a bi-colored graph $G$ on $n$ vertices and $k$ colors such that for every vertex coloring $c$ of $G$, $\omega(c) = 1$ if $c$ is constant, and $\omega(c) = 0$ otherwise?

More questions of this flavor can be found in [KGS19].

**References**