21-355 **Advanced Calculus I** **Fall 2001**

Assignment 1 (Revised 9/10/01)
Due on Monday, September 17

1. Let $\alpha \in \mathbb{R}$ be given and put $S = \{x \in \mathbb{Q} : x < \alpha\}$ be given. Show that $\sup(S) = \alpha$.

2. Use the definition of limit to show that $\dfrac{5}{n^2} \to 0$ as $n \to \infty$.

3.* Use the definition of limit to show that $\dfrac{2n}{n+1} \to 2$ as $n \to \infty$.

4. Let $\{x_n\}_{n=1}^{\infty}$ be a real sequence and $l \in \mathbb{R}\backslash\{0\}$ be given. Assume that $x_n \neq 0$ for all $n \in \mathbb{N}$ and that $x_n \to l$ as $n \to \infty$. Show that $\dfrac{1}{x_n} \to \dfrac{1}{l}$ as $n \to \infty$.

5.* Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be real sequences. Show that if $\{x_n\}_{n=1}^{\infty}$ is bounded and $y_n \to 0$ as $n \to \infty$ then $x_n y_n \to 0$ as $n \to \infty$.

6.* Let $\{x_n\}_{n=1}^{\infty}$ be a real sequence and $l \in \mathbb{R}$ be given. Define the real sequence $\{y_n\}_{n=1}^{\infty}$ by

$$y_n = \frac{1}{n}\sum_{k=1}^{n} x_k \quad \text{for all } n \in \mathbb{N}.$$

Show that if $x_n \to l$ as $n \to \infty$ then $y_n \to l$ as $n \to \infty$.

7. Determine whether or nor $\{\sqrt{n^2+n} - n\}_{n=1}^{\infty}$ is convergent. If it is convergent find the limit.

8.* Let $l \in \mathbb{R}$ be given. Show that there is a sequence $\{r_n\}_{n=1}^{\infty}$ such that $r_n \in \mathbb{Q}$ for every $n \in \mathbb{N}$ and $r_n \to l$ as $n \to \infty$.

* Problems marked with an asterisk should be written up and handed in.