I. The Real Number System

$\mathbb{R}$ denotes the set of real numbers.

$\mathbb{Q}$ denotes the set of rational numbers.

$\mathbb{Z}$ denotes the set of integers.

$\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N} = \{1, 2, 3 \ldots\}$.

I.1 Well-Ordering Principle: Every nonempty subset of $\mathbb{N}$ has a smallest element.

I.2 Principles of Induction: Let S be a subset of $\mathbb{N}$ such that
(i) $1 \in S$, and
(ii) $n + 1 \in S$ for every $n \in S$.
Then $S = \mathbb{N}$.

I.3 Completeness of $\mathbb{R}$: Every nonempty subset of $\mathbb{R}$ that is bounded above has a least upper bound. Every nonempty subset of $\mathbb{R}$ that is bounded below has a greatest lower bound.

I.4 Proposition: Every nonempty finite subset of $\mathbb{R}$ has a largest element and a smallest element.

I.5 Archimedean Property of $\mathbb{R}$: Let $x \in \mathbb{R}$ be given. There exist $n \in \mathbb{Z}$ with $n > x$.

I.6 Proposition: $\mathbb{Q}$ is countable.

I.7 Proposition: For every $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$, the interval (α, β) is uncountable.

I.8 Density of $\mathbb{Q}$ in $\mathbb{R}$: Let $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$ be given. There is a rational number r with $\alpha < r < \beta$.

I.9 Corollary: For every $\alpha, \beta \in \mathbb{R}$ with $\alpha < \beta$, the set $\{r \in \mathbb{Q} : \alpha < r < \beta\}$ is infinite.