

21-127 : Concepts of Mathematics

Summer I 2012

Instructor: Will Gunther
Email: wgunther@math.cmu.edu
Office: Wean 6211
Office Hours: Monday: 3pm-4pm
Tuesday: 3pm-4pm
Wednesday: 5pm-6pm
Thursday: 11:30am-12:30pm
Friday 3pm-4pm
Sundays: 2pm-4pm
(*or by appointment*)
Course Dates: May 21-June 29, 2010
Lecture Times: Weekdays 1:30-2:50
Lecture Place: Wean 4623
Textbook: None required
Suggested: See course websites.

Introduction: To understand what this course is, one must first have an appreciation for what mathematics is. Mathematics is not manipulations of symbols and facts done in the correct order to facilitate a desirable effect on your report card, although this may be what most of you have encountered in the past. It is much more.

The word mathematics comes from the Greek verb “to learn.” As humans, we all have an enormous capacity for reasoning and abstraction. Reasoning is how we make sense of the world and how we properly navigate it. Abstraction is our main tool to make this process easier, where concepts and ideas are organized into discrete components. For example, the word “chair” creates an image in your head which characterizes concept of a chair; thereby, thousands of chairs which you have encountered and used are condensed in one abstract image.

This course is called the Concepts of Mathematics. It is a fearful title to many. But, in light of the last paragraph, I view this course as a exercise of our humanity. In this course we will accomplish three primary tasks.

The first is the namesake of the course, and which utilizes our capacity of abstraction: concepts. You will be introduced to difference abstract concepts which will help us reason very broadly. Without these concepts, our life might be incredibly more difficult. For example, suppose we were not able to see that blue chairs and red chairs were both included in the broader category of chairs. Then, if you reasoned that red chairs have four legs, one would have to do the same work reasoning it for blue chairs. We want to avoid this extra work.

Our second task is to exercise our capacity for reasoning. We all have the ability to reason. We often make mistakes in this reasoning. By exercising our reasoning, we hope to lower the amount of mistakes we make in reasoning and give us tools and intuition to help check our reasoning. The most important of these tools is the notion of a proof. A proof is simply an argument that acts as a “certificate” of correct reasoning.

The third task corresponds to a capacity I have not yet mentioned: knowledge. Knowledge allows us to reason much more quickly by using facts that were previously reasoned and certified, not just by ourself but also by others. Such a fact is called a theorem. In this course, you will learn many theorems. Using these theorems, one can strive to reason about much most advanced things without having to “reinvent the wheel.”

Prerequisites: This course is an introductory course. Therefore, there are no officially listed prerequisites. But, a good grasp of algebra is necessary. A high school level pre-calculus course is sufficient.

Course Objectives: By the end of the course, you are expected to have gained the ability to:

- Understand, Analyze, and Express thoughts phrased in mathematical language and syntax.
- Organize and discuss ideas involving mathematics.
- Understand and Construct proofs using a variety of proof techniques, including direct and indirect proofs and mathematical induction.

- Create, Explain, and Critique proofs involving a broad range of basic mathematical topics, including logic, set theory, and discrete math.
- Identify, Describe, and Prove particular canonical theorems in a broad range of basic mathematical topics, including Cantor's Theorem, Fermat's Little Theorem, Pigeon-Hole Principal, Five-Color Theorem.
- Illustrate different concepts with very specific mathematical examples.

Course Topics: There are three main topics that will be covered in the course.

- *Logic* - First two weeks. This includes: notation, connectives, basic proofs, basic semantics, algebraic properties, proof techniques, induction, basic set theory.
- *Set theory* - Second two weeks. This includes: properties, basic results, relations, order relations, induction and well ordering, equivalence relations, properties of relations, functions, cardinality.
- *Discrete Mathematics* - Third two weeks. This includes: basic counting, stars and bars, multinomials, combinatorial proofs, inclusion-exclusion, pigeon-hole principal, basic graph theory, connectivity, graph coloring.

For more detail on the course schedule and topics, see the course website.

Course Website There is a website for the course. It is <http://math.cmu.edu/~wgunther/127m12/>. It is important to look at the course website regularly, as that is where notes and homeworks will be posted.

Homework: There will be two homeworks a week. One will be Tuesday, and the other Friday. The last homework of the semester will be due on the Wednesday instead of the Tuesday. Assignments will be posted on the course website.

You are free (in fact, encouraged) to work together on the homework. There are two caveats. Firstly, all collaborators must be cited. In addition, collaboration must follow the "white board policy": no permanent notes should be taken when discussing a problem with another person. For example, you can completely solve a problem together at a white board, but then the board must be erased before you write your solutions separately. Collaboration without following these two guidelines constitutes cheating.

Because both Exam 1 and 2 occur on a Monday, there will not be homework on the following Tuesday. This means the Friday homework that week will be particularly long. When a homework is posted, it will be announced in class. The homework will be written in such a way that the problems will follow our progress over the week. This makes them easy to do as we learn the material. They are written with the expectation that you do a few problems everyday after class. They will be too long to try to accomplish the day before they are due.

Homework is worth 40% of the course grade, twice any exam, thus doing well on them is very important for your grade. As you would imagine, spending a lot of time on the homework will also very very good practice for the exam. There are 8 homeworks. They will not be equally weighted. Point values will be assessed to each question, which will reflect how difficult that problem is (or how difficult I think it is).

Exams: In all there are 3 exams. None will be purposefully cumulative, but as with any class in mathematics, material and skills are built throughout the course, and falling behind is a very bad thing.

They are all equally worth one-fifth of your grade.

In the event you cannot make the exam time, you must notify me with advance notice (at least 1 full week) to schedule a makeup.

The structure of the exam may not be something you are used to, but the intent is that it will test you knowledge and encourage incremental studying with the least amount of stress. Each exam will have three parts. The first part will be short answer; these will be easy questions, which will essentially be to know definitions and easy calculations. The second part will be the proof technique part; these will be difficult questions, designed to test if you know how to do certain "types" of proofs. In far advance of the test, you will be given a list of questions, and questions from this part will be taken directly from this list. The third part will be the creativity part; these questions will be of less difficulty than homework questions, but no advance notice is given onto what the questions will be.

Extra Credit: The extra credit will be a project in which you research a topic in mathematics. If you wish to do the extra credit, you must talk with me before the second midterm. There, I will give specifications on what your project can look like and when it is due. Furthermore, I must approve your topic. You cannot do a project unless you discussed it with me before this date.

Grades: Your grade will be based on all the above mentioned work, namely homework and exams. You are more than welcomed to discuss with me if you feel any of this work has been mis-graded or you believe you deserve more credit. Here is roughly how your grade will be calculated:

Homework:	40%
Midterm 1:	20%
Midterm 2:	20%
Midterm 3:	20%
Extra-Credit:	3%

The grades will be assigned on the standard scale:

A	90-100%
B	80-89%
C	70-79%
D	60-69%
R	< 60%

If necessary, these cut offs might be changed at the end of the semester, but only for your benefit.

Course Policies

- **Test Resource Policy:** Outside resources are not permitted on tests. This includes any electronics, books, notes, etc.
- **Homework Resource Policy:** Because people often benefit from seeing many different sources of material, you are welcome to consult different resources not listed on the course website to study and do homework. Such consultation will not be necessary, and all assignments and tests are written to be done without any resources except the class notes. You may not try to find and use an outside source just to answer to a particular problem. For example:
 - Legal Use of Outside Resources: You are stuck on a particular homework problem. Because you are stuck, you realize you may have not known that topic as well as you thought you did. The sources I have provided don't seem to be explaining it to your satisfaction, so you consult an outside source, which clears up your confusion and allows you to do the problem.
 - Illegal Use of Outside Resources: You are, as above, stuck on a particular homework problem. You try searching the question on a search engine, until you find a source which provides an answer to the question. You then paraphrase or copy the answer provided to be turned in.
- **Attendance Policy:** Attendance is required. If you miss a class, you must send me an email with explanation.
- **Academic Honesty:** All work handed in by you, whether in class or homework, must be the work of yourself and no one else. This will be strictly enforced. The penalty for any violation will be at least a 0 on that assignment.
- **Collaboration Policy:** You are encouraged to collaborate. Productive collaboration is a necessary skill to acquire. But, no permanent record of the discussions are allowed (we call this the *White Board Policy*). Moreover, you must cite all collaborators. The penalty for any violation will be at least a 0 on that assignment. For example:
 - Legal Collaboration: You and two friends discuss a problem, all making notes only on a white board. Once you feel like you understand the problem, the white board is erased, and you leave with no permanent notes taken from the meeting. Later, you write up the solution on your own.
 - Illegal Collaboration: You and a friend discuss a problem. You figure it out together, and then compare notes and consult each other while writing up your solutions.
- **Special Needs:** If you have documentation supporting the needs for special accommodations (extra time on tests, special seating, etc) then you must present it to me during the first week of class. I will assist with any reasonable requests.
- **Syllabus Changes:** I reserve the right to make any changes to this syllabus during the course of the semester to make the grading more fair or the course more productive.

Course Advice

- If you do not understand something in class, please ask. Odd are, if you are confused then others in the class are also confused. You are also free to discuss any confusions or problems during my office hours.
- Do not wait till the last minute for anything. The homework will take time. Exams will creep up on you. Time is not a luxury we have in a summer class.
- My office hours will be 6 days a week. Please come to them if you have any concerns or need any help with the course. You can also come if you just want to say hi. If you cannot make any of the times and you'd like to meet with me, send me an email and we meet at a time when you are available.

Important Dates:

May 21: Class Begin
May 25: Drop Deadline
May 28: Memorial Day; No Classes
June 1 Exam 1
June 15 Exam 2
June 28: Last Day of Class. Withdrawal Deadline
June 29: Exam 3