Quiz 15

Problem.

1. $\int \frac{x}{x-6}\,dx$

Bonus. $\int \arctan\left(\frac{1}{x}\right)dx$

Solution.

1. We do the substitution $u = x - 6$. Then $du = dx$ and $x = u + 6$. Thus we have:

$$\begin{aligned}
\int \frac{x}{x-6}\,dx &= \int \frac{u+6}{u}du \\
&= \int \left(\frac{u}{u} + \frac{6}{u}\right)du \\
&= \int \left(1 + \frac{6}{u}\right)du \\
&= u + \ln|u| + C \\
&= x - 6 + \ln|x-6| + C \\
&= x + \ln|x-6| + C
\end{aligned}$$

Bonus We begin with parts:

$$\int \arctan\left(\frac{1}{x}\right)dx$$

$$\begin{aligned}
u &= \arctan\left(\tfrac{1}{x}\right) & dv &= dx \\
du &= \tfrac{1}{1+(1/x)^2}\cdot\left(-\tfrac{1}{x^2}\right) & v &= x
\end{aligned}$$

$$\begin{aligned}
&= \arctan\left(\frac{1}{x}\right)\cdot x - \int\left(\frac{1}{1+(1/x)^2}\cdot\left(-\frac{x}{x^2}\right)\right)dx \\
&= x\arctan\left(\frac{1}{x}\right) + \int\left(\frac{x}{x^2+1}\right)dx
\end{aligned}$$

$$\begin{aligned}
u &= x^2 + 1 \\
du &= 2x\cdot dx \\
\tfrac{du}{2} &= x\cdot dx
\end{aligned}$$

$$\begin{aligned}
&= x\arctan\left(\frac{1}{x}\right) + \frac{1}{2}\int\left(\frac{1}{u}\right)du \\
&= x\arctan\left(\frac{1}{x}\right) + \frac{\ln|x^2+1|}{2} + C
\end{aligned}$$