Fall 2026 **21371 - Functions of a Complex Variable** **CMU**

Lecture: MWF 10:00 am – 10:50 pm, Wean Hall 8201

Lecturer: Tomasz Tkocz, Wean Hall 8117, ttkocz@math.cmu.edu

Office Hours: Monday 11:00 – 11:50 am, or by email appointment

Course website: Canvas and/or http://math.cmu.edu/~ttkocz

Course description: This course is a rigorous introduction to complex analysis, starting from the notion of complex differentiation, entering a marvellous world, full of wonderful insights, where functions differentiable once are automatically indefinitely differentiable. The highlights include slick proofs of the fundamental theorem of algebra, the Riemann zeta function central in the theory of prime numbers, the Fourier transform, conformal mappings and (un)expected applications of these in other areas of mathematics.

Prerequisites: complex numbers, certain maturity with real analysis (for instance within the scope of a solid undergraduate class such as Principles of Real Analysis I and II)

Literature:

- Ahlfors, L., Complex Analysis : An Introduction to The Theory of Analytic Functions of One Complex Variable. *McGraw-Hill*, 1979.
- Bak, J., Newman, D. J., Complex Analysis. Undergraduate Texts in Mathematics, *Springer*, 2010.
- Stein, E., Shakarchi, R., Complex Analysis. *Princeton University Press*, 2003.
- Davenport, H., Multiplicative Number Theory. Graduate Texts in Mathematics Vol. 74, *Springer*, 2000.
- Newman, D. J., Analytic Number Theory. Graduate Texts in Mathematics Vol. 177, *Springer*, 1998.

Course content: functions on the complex plane, complex derivative, the Cauchy-Riemann equations, integration along curves, Goursat's theorem, Cauchy's theorem and integral formulas, Morera's theorem, Schwarz reflection principle, zeros and poles, the residue formula, singularities and meromorphic functions, the argument principle and applications, Rouché's theorem, the open mapping theorem, the maximum principle, the Fourier transform, Paley-Wiener type theorems, Jensen's formula, Hadamard's factorisation theorem, the gamma and zeta functions, conformal mappings, the prime number theorem, elliptic functions (time permitting)

Learning objectives: Students should

- gain understanding of basic properties of analytic functions
- advance their insight into the interplay between algebraic, geometric and analytic ideas
- develop an improved ability and use the methods and results of complex analysis, with applications in other areas, particularly algebra, number theory, combinatorics, geometry, among others

Course format: This is an in person class. You are expected to fully participate in class, viz. please ask and answer questions, initiate or participate in discussions. We follow rather closely the Stein-Shakarchi textbook.

Homework: There is no assessed homework. I will maintain a list of practice problems (think of them as suggested homework). This is the essential part of the learning process in this course. Simply listening in class or reading texts is not sufficient. Understanding mathematics requires practice.

Exams: There will be 4 in-class pop up quizzes (15min) throughout the semester (based on the practice problems and the lecture material), and a final presentation-style project. *No* final exam. Plagiarism and cheating are not tolerated.

Grades: The midterm grade will be based solely on the pop up quizzes. The final grade will be based on the quizzes and the final project, computed as a weighted average:

$$50\% \textbf{ Quizzes} + 50\% \textbf{ Project}$$

Rough guide on "score" $\to$ "grade" map: `https://en.wikipedia.org/wiki/Academic_grading_in_the_United_States` (but the grades will be "curved" if needed)

The sweeping development of mathematics during the last two centuries is due in large part to the introduction of complex numbers; paradoxically, this is based on the seemingly absurd notion that there are numbers whose squares are negative.

–E. Borel, 1952