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# DID FAULTY MATHEMATICAL MODELS CAUSE THE FINANCIAL FIASCO?

## BLAME IT ON GAUSSIAN COPULA



BY STEVEN E. SHREVE

*“As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.”*

– Albert Einstein

OVER THE PAST 20 YEARS THE PRACTICE OF FINANCE HAS BEEN REVOLUTIONIZED BY THE WIDESPREAD ADOPTION OF MATHEMATICAL MODELS FOR PRICING EVER MORE EXOTIC DERIVATIVE SECURITIES.

As we face the worst financial crisis since the Great Depression, we cannot help but ask what role mathematics played in this. Over the past 20 years the practice of finance has been revolutionized by the widespread adoption of mathematical models for pricing ever more exotic derivative securities. Mortgage-backed securities, which triggered the financial collapse, were priced using the Gaussian copula model. What would Albert Einstein have to say about all this?

I suppose Einstein would remind us that to build a mathematical model, it is necessary to make assumptions. At best, these assumptions are idealizations of reality. The art of model building is to choose assumptions so

that the model is unencumbered by less relevant considerations in order to illuminate more important phenomena. A good model provides insight and a guide to action. A good user of a model understands its limitations. We have seen all this play out in the financial markets.

**Replication.** The granddaddy of financial models is due to Black, Merton and Scholes. This model begins with the assumption that a stock price evolves continuously in time and has a constant *volatility*, a parameter characterizing the risk of the stock. A European call on such a stock is the option to buy a share of the stock at a future “expiration” date in exchange for a payment set at the initial time but paid at expiration. Since the owner of the call has a potential gain at expiration, the call has a positive initial value or *price*

Black and Scholes provided a formula for the price of the European call. Merton provided the replication argument we now use to understand it. Consider an investor who invests in the stock and borrows from a money-market account to finance this. It turns out that if the investor can trade continuously, then she can start with a certain initial capital and trade in such a way that the value of her portfolio at expiration agrees with the payoff of the call. The initial capital that permits this must be the initial price of the call, and it is given by the Black-Scholes formula.

The Black-Scholes-Merton analysis contains the insight of pricing the call by replication rather than just computing the expected value of its payoff. The flip side of this is hedging. If one owns the call and uses the negative of the

replicating strategy, then one has a hedged position. Any loss in the value of the call will be offset by a gain in the value of the portfolio held by this negative replication, and vice versa. An investment bank performing intermediation among parties must hold assets for a time, and during that time hedging protects the bank against loss. The replication argument involves a delicate interplay between the sensitivity of the call price to movements in the stock price and to the ever-decreasing time to expiration, and this balancing act is where the stock volatility matters.

**Risk-Neutral Pricing.** In the early 1980s, M. Harrison, D. Kreps, and S. Pliska developed a *risk-neutral pricing formula* that greatly extended the applicability of pricing by replication. They pointed out that once a model is built, one can change the probability measure on the space on which the stock price is defined so that it has mean rate of return equal to the interest rate. This is akin to building a model based on tosses of a fair coin, and then pretending for computational purposes that the coin is biased. Under this so-called risk-neutral measure, both the stock and the money-market account held by the portfolio replicating a call have mean rate of return equal to the interest rate, and so the portfolio itself has this mean rate of return. Therefore, the initial value of the portfolio, which is the Black-Scholes price of the call, can be obtained by discounting the call payoff at the interest rate and taking the expected value under the risk-neutral measure.

One can thus build a model with multiple primary assets, change to a risk-neutral measure,

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and then price all derivative securities using the risk-neutral pricing formula. Harrison and Pliska showed that the risk-neutral measure is unique if and only if every derivative security can be replicated. In such a situation, risk-neutral pricing is justified.

Although based on a theory replete with assumptions, in practice the concept of a risk-neutral pricing was quickly cut loose from its moorings. In markets in which a contract having a random payoff was to be priced, assumptions about the distribution of the payoff were made and the initial price of the contract was computed by discounting the payoff and taking the expectation using this distribution. Parameters in the assumed distribution were tuned so that the model prices fit the market prices for contracts whose market prices could be observed. The resulting distribution was called the risk-neutral measure, with little thought as to whether it was unique and whether something like the delicate balancing act behind the Black-Scholes-Merton replication argument would work.

**The Gaussian Copula.** Replication is not possible in the market for mortgage-backed securities. Consider a pool that holds 100 identical mortgages. A “super-senior” tranche of this pool might consist of the 80 last-to-default mortgages. An investor who purchases the super-senior tranche will receive the interest and principal payments from 80 mortgages. This investor is unaffected by defaults in the pool until the lower tranches, which comprise the first 20 mortgages to default, have been wiped

out. Only the 21st and later defaults will be felt by the super-senior investor.

It is tempting to price a super-senior tranche by finding a probability distribution for the payments that will be received, discounting them to allow for the time-value of money, and taking an expected value. This is what is attempted by the Gaussian copula model, which became the industry standard for these products. But determining the distribution of the payments is notoriously difficult because the probability that there will be 21 or more defaults depends critically on the assumption about the correlation of default among the different mortgages. If all can be brought down by a single event, such as a decline in housing prices, then the super-senior tranche is much riskier than if 21 independent failures must occur in order to hurt the super-senior investor. The correlation parameter values required by the Gaussian copula model to get model prices to agree with market prices have turned out to be woefully wrong.

More worrisome than finding the right distribution is the fact that the Gaussian copula model cannot guide replication. It is a static model. It cannot capture a delicate tradeoff that involves time to expiration because the model does not evolve in time. This model is risk-neutral pricing cut loose from its moorings. This was obvious to anyone who examined the model. But curiously, despite this shortcoming, ratings agencies, risk managers and traders treated the Gaussian copula prices as completely reliable, just as reliable as if replication were possible. As we are learning, multi-billion dollar positions were taken

based on the model. When prices collapsed, banks had no hedges to protect themselves.

**Conclusion.** The best reason to understand the models used in any practical endeavor is in order to know their limitations. The successes of quantitative methods in finance in the 1980s and 1990s created an aura of invincibility. But even so, it used to be customary to “stress test” models, to see how sensitive their outputs were to slight changes in the never-quite-correct assumptions on which they were based. Perhaps that was discontinued because the Gaussian copula model could not pass the tests. The sensitivity of the model to the correlation assumption was well known. The inability of the model to provide hedging guidance was understood. There is no way to hedge the risk from default in a tranche of a mortgage-backed security, and the model is honest about that.

All of this brings us to the subject of discriminating use of mathematical models and how we got into the present financial mess. At least part of the answer is that the careful people were told, “Everyone is making money, so why aren’t you?” After a few years, “model risk” was no longer an acceptable answer. It should be once again.

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