

Some HW problems in model theory (April 22, 2002).

1. T_{ord} has a universal model of cardinality \beth_ω .
2. Suppose $G \subseteq \mathbb{C}^n$ is a definable group in the field of complex numbers $\langle \mathbb{C}, +, \cdot \rangle$. prove that there is no $\{H_n \mid n < \omega\}$ infinite decreasing chain of definable subgroups of G .
3. Prove T_{ord} is not stable.
4. Show that all 3 theories from Example I.2.46 are \aleph_0 -stable.
5. T_{ind} is unstable.
6. T_{ACF_p} is \aleph_0 -stable.
7. Suppose $M \models T_{ord}$ is saturated. Show that for every $A \subseteq |M|$ such that $|A| < ||M||$ there exists $b \in |M|$ such that $M \models [a < b]$ for every $a \in A$.
8. Use the previous exercise to show that T_{ord} does not have a saturated model of cardinality \beth_ω .
9. (hard) Show that if T is \aleph_0 -stable then union of a chain of saturated models is saturated.