## 21-240: Final exam Topics List

YOU SHOULD ALSO KNOW EVERYTHING FROM THE PREVIOUS TOPICS LISTS.

Definitions (this list is not necessarily complete! You should know all of the definitions in the sections we covered, these are the most important ones): Complex eigenvalues and eigenvectors, inner product, dot product, length of a vector, outer product, norm, distance between vectors, distance between a point and a subspace, orthogonal, orthogonal complement, angles, orthogonal projection, orthonormal, least squares approximation/solution, symmetric matrix, orthogonally diagonalizable.

## **Topics:**

- Diagonal matrix representation using basis of eigenvectors
- Every n by n matrix has n eigenvalues with multiplicity if allowing complex eigenvalues
- Computing complex eigenvalues and eigenvectors
- Computing length of a vector
- Distance between points
- Orthogonal vectors and sets of orthogonal vectors
- The orthogonal complement of the row space is the null space
- The orthogonal complement of the column space is the null space of  $A^T$ .
- Angles via dot product
- Showing a vector is in the orthogonal complement of a subspace
- Computing orthogonal projections
- Normalizing vectors
- Matrices with orthonormal columns

- The orthogonal decomposition theorem
- Best approximation theorem
- Orthonormal bases
- Least squares approximation
- Properties of symmetric matrices
- The real spectral theorem statement
- Orthogonally diagonalizable iff symmetric.