Math 301: Homework 4

Due Wednesday Wednesday October 4 at noon

1. Let G be a graph on n vertices with e edges. We showed in class that the maxcut of G has size at least $\frac{e}{2}$. Be smarter than I was in class, and show that the maxcut has size at least $\frac{n}{2n-1}e$.

2. Let G be a bipartite graph on n vertices. For each vertex $v \in V(G)$, let $L(v)$ be a list of colors associated to v of size $\lfloor \log_2 n \rfloor + 1$. Show that it is possible to choose for each vertex v a color from $L(v)$ such that no edge has two endpoints that are the same color.

3. We use the notation $\text{ex}(n, C_4)$ to denote that maximum number of edges in an n-vertex graph that does not contain a cycle on 4 vertices.

   a Note that the number of C_4s in a complete graph on n vertices is $\binom{n}{4}\binom{4}{2}\frac{1}{2}$.

   b Use expectation to show that $\text{ex}(n, C_4) = \Omega(n)$.

   c Use the alteration method to show that $\text{ex}(n, C_4) = \Omega\left(n^{4/3}\right)$.