



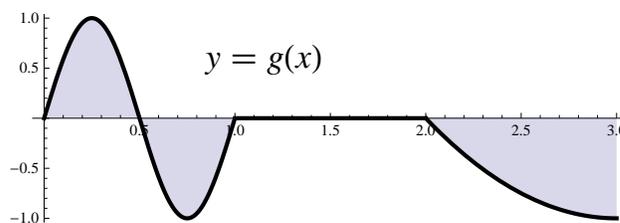
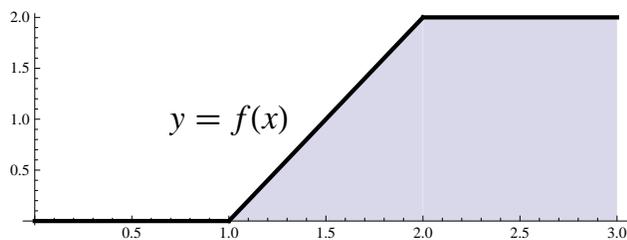
University of California, San Diego
Department of Mathematics

Instructions

1. Write your *Name*, *PID*, *Section Number* and the *Version* of your exam on the front of your Blue Book.
2. No calculators or other electronic devices are allowed during this exam.
3. You may use one page of notes, but no books or other assistance during this exam.
4. Read each question carefully, and answer each question completely.
5. Write your solutions clearly in your Blue Book
 - (a) Carefully indicate the number and letter of each question.
 - (b) Present your answers in the same order they appear in the exam.
 - (c) Start each question on a new page.
6. Show all of your work; no credit will be given for unsupported answers.

0. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. The following are graphs of the functions f and g over the interval $[0, 3]$.



Assume $\int_0^1 g(x) dx = 0$ and $\int_2^3 g(x) dx = -\frac{2}{3}$.

- (a) (4 points) Compute $\int_0^3 (2f(x) + 3g(x)) dx$.
 - (b) (3 points) Suppose f is an odd function. Compute $\int_{-2}^0 f(x) dx$.
 - (c) (3 points) Suppose $G'(x) = g(x)$ and $G(0) = 1$. Compute $G(3)$.
2. (10 points) Compute the following indefinite integral.

$$\int \frac{x^2 + 3x + 1}{x} dx$$

3. (10 points) Compute the derivative:

$$\frac{d}{dx} \int_3^{\sin(x)} e^{\sqrt{t}} dt.$$

4. (9 points) Compute the indefinite integral:

$$\int x^3 \ln x \, dx$$

5. (10 points) Compute the definite integral:

$$\int_0^1 \frac{x}{\sqrt{4+x^2}} \, dx$$

6. (10 points) Use the method of partial fractions to compute the indefinite integral:

$$\int \frac{x}{(x-1)(x+1)^2} \, dx \quad (\text{Hint: } \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2})$$

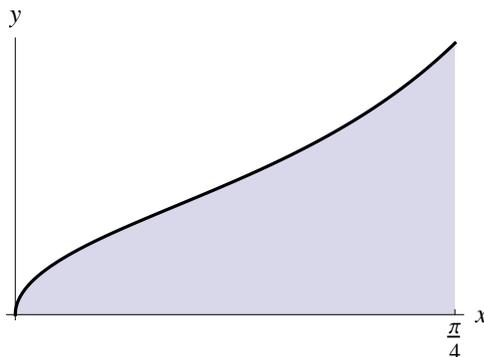
7. (a) (5 points) The following improper integral converges. Compute its value:

$$\int_2^5 \frac{1}{\sqrt{x-2}} \, dx$$

- (b) (5 points) Use the inequality $\ln(x) \leq x$ for all $x > 1$ to show that the following improper integral converges:

$$\int_1^{\infty} \frac{\ln x}{x^3 + 1} \, dx.$$

8. (10 points) The shaded region below is bounded by $y = 0$, $x = \frac{\pi}{4}$, and $y = \sqrt{\tan x \sec^2 x}$.



Calculate the volume of the solid formed by rotating the region about the x -axis.

(Hint: $\sec(\pi/4) = \sqrt{2}$.)

9. (10 points) Find the values of k such that $y(x) = e^{kx}$ is the solution to

$$\frac{d^2 y}{dx^2} = 4y.$$

10. (10 points) Solve the initial value problem (IVP):

$$\frac{dy}{dt} = -t(y-2), \quad \text{with } y(0) = 5.$$

(This exam is worth 100 points.)