Probability | Victor Xu

Probabilistic Strategies

Western PA ARML Practice | *April 3, 2016*

1 Problems

1. You roll two 6-sided dice. What's the probability of rolling at least one 6?

2. Two teams are playing each other in a best-to-3 series. Each team has a 50% chance of winning any individual game. What's the probability that the series will go to 5 games?

3. You are playing a game against a friend: you and your friend draw the top card off of a deck of cards, and you win if you and your friend both drew cards of the same suit. A deck of 52 cards is shuffled and you start playing, without shuffling the used cards back into the deck. What's the probability that you win the 5^{th} game?

4. You roll two 6-sided dice. What's the probability that the sum of the numbers you rolled is even?

5. You're playing 3-card anti-poker, where the goal is to not make good poker hands. What's the probability that a random hand of 3 cards from a 52 card deck has cards of not all the same suit and does not contain a pair?

2 More Problems

A few of these problems use new ideas that don't show up in the problems above. If you get stuck, don't hesitate to ask a friend or a coach for help!

1. (2006 AMC 10B Problem 17) Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

2. You have a bag with 5 green balls, 2 yellow balls, and 6 red balls. You take balls out without putting them back in the bag, until there are no balls left. What's the probability that the third to last ball you draw is green or yellow?

3. What's the probability that a randomly chosen integer between 1 and 1000 inclusive contains the digit 0?

4. You're playing some games of rock-paper-scissors with a mind-reading rock-hating friend who refuses to play rock. To ensure fairness, your friend is not allowed to use his/her mind reading powers during the games, but he/she will know your strategy. What's your best strategy?

5. You're repeatedly flipping a fair coin. You get a point if you flipped heads, and you lose a point if you flipped tails. You win if you get 10 points, and you lose if you get -5 points. What's the probability that you win?

6. You're playing a coin flipping game against your friend. You repeatedly flip a fair coin, and you win if at any point the last two flips were Tails and then Heads. Your friend wins if the last two flips at any point are both Heads. What's the probability that you win this game? What about if you win when TTH is flipped and your friend wins when HHH is flipped?

7. (2007 AIME II Problem 10) Let S be a set with six elements. Let P be the set of all subsets of S. Subsets A and B of S, not necessarily distinct, are chosen independently and at random from P. The probability that B is contained in at least one of A or $S \setminus A$ is $\frac{m}{n^r}$, where m, n, and r are positive integers, n is prime, and m and n are relatively prime. Find $m + n + r$. (The set $S \setminus A$ is the set of all elements of S which are not in A.)

8. You're playing a game that involves scoring points by collecting cubes of the same color. Every time you receive a cube, there is a $1/2$ chance that it is white, a $1/3$ chance that it is blue, and a $1/6$ chance that it is a red cube. Your total score at the end of the game is the number of white cubes you have plus the product of the number of red and the number of blue cubes you have collected. For example, if you have 5 white cubes, 3 blue cubes, and 2 red cubes, your score would be $5 + 3 \cdot 2 = 11$ points. What is the expected value of the number of points you will have, if you receive 10 cubes over the course of the game? (The expected value of a random value is the "mean" value, for example, the expected value of a die roll is 3.5.)