Number Theory — Misha Lavrov

Divisors and Factors

Western PA ARML Practice — *January 25, 2015*

Warm-up

1. (AMC 10 2008) Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

Problems

1. How many divisors does 10000 have? (That is, how many numbers between 1 and 10000 divide 10000 evenly?)
2. How many of the numbers between 1 and 100 have exactly 3 divisors?
3. Find the smallest integer with exactly 10 divisors.
4. What is the sum of all the divisors of 10000? (Try to find a way to do this without actually adding up a bunch of numbers.)
5. If the sum of all of the divisors of n (including n itself) is 91, what is n?
6. For what values of n will a regular n-sided polygon have angles whose measure (in degrees) is an integer?
7. A triple of positive integers (x, y, z) is called a *Pythagorean triple* if $x^2 + y^2 = z^2$. Find all Pythagorean triples where $x = 8$ or $x = 9$. (Don't assume that $x < y$.)
8. Find all pairs of numbers (x, y) such that the GCD of x and y is 12, and the LCM of x and y is 180.
9. Find all pairs of numbers (x, y) such that the GCD of x and y is 12, and the product of x and y is 180.
10. Find the number of zeroes at the end of $100! = 1 \times 2 \times 3 \times \cdots \times 100$.