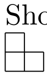
Chess | Misha Lavrov

Assorted problems about chess

Western PA ARML Practice | *December 7, 2014*

Tiling problems

1. Can an 8×8 chessboard with two opposite corners removed be covered with 31 1×2 tiles?
2. For which m, n can an $m \times n$ chessboard be covered with non-overlapping 4×1 tiles? What about $a \times 1$ tiles?
3. Show that a $2^n \times 2^n$ chessboard with any square removed can be covered with non-overlapping  -shaped tiles.
4. What is the largest number of non-overlapping 5-square tiles in the shape of a plus sign that can be placed on an 8×8 chessboard?

Chess piece problems

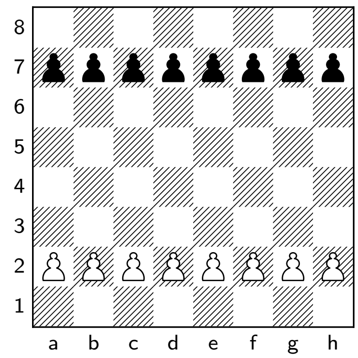
1. A Queen in chess attacks all spaces vertically, horizontally, or diagonally in a straight line. Place 8 Queens on an 8×8 chessboard so that none of them attack each other.
2. A Knight attacks any space exactly $\sqrt{5}$ units away (that is, 2 spaces in one direction and 1 space in a perpendicular direction). What is the largest number of Knights that can be placed on an 8×8 chessboard so that none of them attack each other?
3. A Bishop attacks all spaces diagonally in a straight line. What is the largest number of Bishops that can be placed on an 8×8 chessboard so that none of them attack each other?
4. A King attacks all spaces adjacent to it vertically, horizontally, or diagonally.

   On a *toroidal* chessboard, the top and bottom row are adjacent, as are the leftmost and rightmost column. What is the largest number of Kings that can be placed on a 5×5 toroidal chessboard so that none of them attack each other?
5. In 1979, László Lovász proved that at most C^n Kings can be placed on an n-dimensional $5 \times 5 \times \cdots \times 5$ toroidal chessboard so that none of them attack each other, where C^2 is the answer to the previous problem. This is very hard. What's less hard is the following question: how can you achieve the bound of C^n for any *even* dimension n?
6. A chess piece I just invented, which I'm calling the Emperor, attacks all spaces that a King attacks, and also attacks two spaces away horizontally or vertically. What is the largest number of Emperors that can be placed on an 8×8 chessboard so that none of them attack each other?

   What is the relationship between this problem and the last problem (problem 4) in the first section?

The Pawn Game

In this section, pawns behave slightly differently than usual.



The pawns start in the arrangement above. Pawns can only move forward: for White pawns, this means going to a row with a higher number, and for Black pawns, this means going to a row with a lower number.

Contrary to chess rules, pawns always have the option of moving one space forward (if it is empty), or two spaces forward (if both spaces are empty).

Contrary to chess rules, pawns cannot capture. The game ends when there is no move possible, at which point the player with no move left to make loses.

1. With optimal play, who wins the pawn game: White or Black? (Describe how that side should play to win.)
2. What about on a 7×7 chessboard? On a 9×9 chessboard?
3. In each of these cases, what if pawns can also move backward (with the restriction that they cannot leave the board)?
4. Find a general rule to win the pawn game from any position. Use it to find all winning moves for White in each of the games below: