DIFFERENTIAL EQUATIONS HOMEWORK 4
ADDITIONAL PROBLEMS

2006 SPRING

(1) Consider the predator-prey model

$$\frac{dR}{dt} = 3R - 2RF$$

$$\frac{dF}{dt} = -8F + 2RF$$

(a) Show that this system has exactly one non-zero equilibrium point, (R_0, F_0).

(b) The lines $R = R_0$ and $F = F_0$ divide the phase plane for this system (i.e. the RF-plane) into four regions. Determine the direction of solutions in each of these four regions (e.g. "up-right," "up-left," "down-right," or "down-left"). **Remember:** at the point (r, f) in the RF-plane, the solution curve $(R(t), F(t))$ is traveling in the direction

$$\left(\left.\frac{dR}{dt}\right|_{(r,f)}, \left.\frac{dF}{dt}\right|_{(r,f)} \right).$$

(c) When crossing the line $R = R_0$ solutions must be traveling horizontally in the phase plane. Why is this? What can you say about the direction solutions travel as they cross the line $F = F_0$?

(2) Consider the system of differential equations

$$\frac{dx}{dt} = 2xy$$

$$\frac{dy}{dt} = -3y$$

Notice that the second differential equation does not depend on x. This is an example of a "partially decoupled" system.

(a) Find the solution for the second equation satisfying the initial condition $y(0) = 1$.

(b) Substitute the solution found in 3a for y in the first equation. Solve the resulting equation subject to the initial condition $x(0) = 2$.

(c) Find a solution to the system above that satisfies the initial condition $(x(0), y(0)) = (2, 1)$. [Hint: This should be easy once you complete 3a and 3b.]