21-260 **Differential Equations** **D. Handron**

Review #3

1. Consider the following differential equation, which models a damped mass-spring system:

$$y'' + 14y' + 58y = 0.$$

   (a) Classify the system as overdamped, underdamped or critically damped.

   (b) Suppose an external force $F(t) = e^{-3t}$ is applied to the system above. Find the general solution to the resulting non-homogeneous differential equation.

   (c) Identify the transient solution and the steady state solution. Describe the *long term* behavior of the system.

2. The motion of an undamped mass-spring system is described by the differential equation

$$2x'' + 36x = \sin(\omega t).$$

   (a) For what values of ω will the system exhibit resonance?

   (b) Use the method of undetermined coefficients to find a particular solution in the case where ω is *not* the resonant frequency.

3. Let

$$g(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & 2 \le t. \end{cases}$$

   (a) Find $G(s) = \mathcal{L}\{g(t)\}$.

4. Determine the following inverse Laplace transforms:

   (a) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+9}\right\}$

5. A mass-spring-dashpot system can be modeled by the second order equation

$$m\frac{d^2x}{dt^2} = -k_s\frac{dx}{dt} - k_d x,$$

   where m is the mass, k_s is the spring constant and k_d is the damping coefficient.

   (a) A certain system of this type with $m = 1$ can also be modeled by the first order system

$$\begin{bmatrix} x' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}.$$

   What is the spring constant in this system? What is the damping coefficient?

(b) The complex number $-2+i$ is an eigenvalue of the matrix $\begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix}$. An eigenvector corresponding to $-2+i$ is $\begin{bmatrix} 1 \\ -2+i \end{bmatrix}$. What is the general solution?

(c) What is the general solution to the second order equation $\frac{d^2x}{dt^2} = -4\frac{dx}{dt} - 5x$?