21-260 **Differential Equations** **D. Handron**

Review #3

1. Solve the initial value problem

$$\begin{gathered} y'' + 2y' + 5y = 0 \\ y(0) = 2 \\ y'(0) = -1 \end{gathered}$$

using the Laplace transform method.

2. Consider the following differential equation, which models a damped mass-spring system:

$$y'' + 14y' + 58y = 0.$$

(a) Classify the system as overdamped, underdamped or critically damped.

(b) Suppose an external force $F(t) = e^{-3t}$ is applied to the system above. Find the general solution to the resulting non-homogeneous differential equation.

(c) Identify the transient solution and the steady state solution. Describe the *long term* behavior of the system.

3. Consider the system of differential equations,

$$\begin{aligned} \tfrac{dx}{dt} &= 3x - xy \\ \tfrac{dy}{dt} &= -4y + 2xy. \end{aligned}$$

(a) Find the equilibrium points of this system.

(b) Classify the equilibrium point at $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as to type and stability.

4. The motion of an undamped mass-spring system is described by the differential equation

$$2x'' + 36x = \sin(\omega t).$$

(a) For what values of ω will the system exhibit resonance?

(b) Use the method of undetermined coefficients to find a particular solution in the case where ω is *not* the resonant frequency.

5. Let

$$g(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & 2 \le t. \end{cases}$$

(a) Find $G(s) = \mathcal{L}\{g(t)\}$.

(b) Find $\mathcal{L}^{-1}\left\{\dfrac{1-2e^{-(s-5)}+e^{-2(s-5)}}{s-5}\right\}$.

6. Determine the following inverse Laplace transforms:

   (a) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+9}\right\}$

7. Use the Laplace transform method to solve the initial value problem

$$x'' + 2x' + 5x = 0; \quad x(0) = 1, x'(0) = 2.$$

8. A mass-spring-dashpot system can be modeled by the second order equation

$$m\frac{d^2x}{dt^2} = -k_s\frac{dx}{dt} - k_d x,$$

where m is the mass, k_s is the spring constant and k_d is the damping coefficient.

   (a) A certain system of this type with $m = 1$ can also be modeled by the first order system

$$\begin{bmatrix} x' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}.$$

   What is the spring constant in this system? What is the damping coefficient?

   (b) The complex number $-2+i$ is an eigenvalue of the matrix $\begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix}$. An eigenvector corresponding to $-2+i$ is $\begin{bmatrix} 1 \\ -2+i \end{bmatrix}$. What is the general solution?

   (c) What is the general solution to the second order equation $\frac{d^2x}{dt^2} = -4\frac{dx}{dt} - 5x$?