21-260 **Differential Equations** **D. Handron**

Review #3

1. Consider the linear system of differential equations

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x},$$

where

$$A = \begin{bmatrix} 1 & 5 \\ 10 & -4 \end{bmatrix}.$$

(a) Find the eigenvalues of A.

(b) Find the eigenvectors of A.

(c) Solve the initial value problem $\frac{d\mathbf{x}}{dt} = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 0 \\ 9 \end{bmatrix}$.

2. A mass-spring-dashpot system can be modeled by the second order equation

$$m\frac{d^2x}{dt^2} = -k_s\frac{dx}{dt} - k_d x,$$

where m is the mass, k_s is the spring constant and k_d is the damping coefficient.

(a) A certain system of this type with $m = 1$ can also be modeled by the first order system

$$\begin{bmatrix} x' \\ v' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}.$$

What is the spring constant in this system? What is the damping coefficient?

(b) The complex number $-2 + i$ is an eigenvalue of the matrix $\begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix}$. An eigenvector corresponding to $-2 + i$ is $\begin{bmatrix} 1 \\ -2 + i \end{bmatrix}$. What is the general solution?

(c) What is the general solution to the second order equation $\frac{d^2x}{dt^2} = -4\frac{dx}{dt} - 5x$?

3. Consider the system of differential equations,

$$\begin{aligned} \frac{dx}{dt} &= 3x - y \\ \frac{dy}{dt} &= -4y + 2x. \end{aligned}$$

(a) Find the equilibrium points of this system.

(b) Classify the equilibrium point at $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ as to type and stability.

4. Consider the 3-dimensional system of linear differential equations

$$\frac{d\mathbf{x}}{dt} = \begin{bmatrix} x_1{}' \\ x_2{}' \\ x_3{}' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \mathbf{x}.$$

   (a) Find the eigenvalues for the system.

   (b) Find the eigenvectors.

   (c) Find the general solution to the system.

5. (a) Find the general solution to the linear, first-order system

$$\frac{dX}{dt} = \begin{bmatrix} 0 & -12 \\ 2 & -10 \end{bmatrix} X.$$

   (b) Sketch some representative solution curves in the phase plane, and classify the equilibrium point as to type and stability.

6. Consider the linear system

$$\frac{dX}{dt} = \begin{bmatrix} 0 & -2 \\ -8 & 0 \end{bmatrix} X = AX.$$

   (a) Find the general solution for the system using the eigenvalue method.

   (b) Sketch some representative solutions in the phase plane.