21-260 **Differential Equations** **D. Handron**

Exam #1 Review

Closed book and notes; calculators not permitted. Be sure to show all work and explain your reasoning as clearly as possible.

1. Consider the first order differential equation

$$\frac{dy}{dt} = \frac{t}{2y+1}$$

   (a) Find the general solution.

   (b) Find the particular solution with $y(1) = \frac{3}{2}$.

   (c) Is this the unique solution with $y(1) = \frac{3}{2}$? How can you tell?

2. (a) Solve the initial value problem

$$xy' + 3y = x^3, \qquad y(1) = 10$$

   (b) What can you say about the existence and uniqueness of a solution to the above differential equation with initial condition $y(3) = 2$? What about $y(0) = 1$?

3. Sketch the phase line for the differential equation $y' = (y-1)(y+1)(y+2)^2$ Label each critical point as stable, semi-stable or unstable.

4. Consider the differential equation

$$\frac{dy}{dx} = (y-1)(y+2)^2(y^2+1).$$

   (a) Draw the phase line (phase diagram) for this differential equation.

   (b) Let $y(t)$ be the solution satisfying the initial condition $y(0) = 0$. Can the value of $y(t)$ ever be less than -2? Why or why not?

5. Find the general solution for the differential equation

$$\frac{dy}{dx} = \frac{x(y^2+1)}{2y}$$

6. Consider the first order differential equation

$$\frac{dy}{dt} + \sin(t)y = \sin(t)$$

   (a) Find an integrating factor for the equation.

   (b) Find the general solution.

   (c) Find the particular solution with $y(\frac{\pi}{2}) = 0$.

7. Solve the initial value problem

$$\begin{aligned} \tfrac{dy}{dt} &= 3 - 2y \\ y(0) &= 0. \end{aligned}$$

8. An impatient coffee drinker has placed his scalding hot (160°) coffee into a 40° refrigerator. After three minutes, the temperature has fallen to 140°. How long until the temperature of the coffee has fallen to a temperature that is safe to drink (120°). [You may want to use Newton's Law of Cooling: $\frac{dT}{dt} = -k(T - T^*)$. You may express your result in terms of natural logarithms of various numbers.]

9. A vat initially (at $t = 0$) holds 100gal of pure water. A salt solution is added to the vat at a rate of 2gal/min. The concentration of this solution (in oz/gal) varies and is given by $c(t) = \frac{1}{2}t(50 - t)$. The (well mixed) solution is allowed to flow out of the vat at a rate of 4gal/min. Find an expression for the amount of salt in the vat as a function of t.