21-260 Differential Equations

Homework #1

Due Friday 20 January

1.3.2. Determine the order of the differential equation; also state whether the equation is linear or nonlinear.

$$(1+y^2)\frac{d^y}{dt^2}+t\frac{dy}{dt}+y=e^t.$$

1.3.10. Verify that each given function is a solution of the differential equation.

$$y''''+4y'''+3y=t; \qquad y_1(t)=\frac{t}{3}, \quad y_2(t)=e^{-t}+\frac{t}{3}$$

1.3.27. Verify that each given function is a solution of the given partial differential equation.

$$a^2u_{xx}=u_{tt}; \qquad u_1(x,t)=\sin\lambda x\sin\lambda at, \quad u_2(x,t)=\sin(x-at),$$

where λ is a real constant.

1.3.30. Another way to derive the pendulum equation

$$\frac{d^2\theta}{dt^2}+\frac{g}{L}\sin\theta=0 \tag{1}$$

is based on the principle of conservation of energy. Here L is the length of the pendulum, and θ measures the angle the pendulum makes with a vertical line, measured from the downward position.

1. Show that the kinetic energy T of the pendulum in motion is
$$T=\frac{1}{2}mL^2\left(\frac{d\theta}{dt}\right)^2.$$
2. Show that the potential energy V of the pendulum, relative to its rest position, is
$$V=mgL(1-\cos\theta).$$
3. By the principle of conservation of energy, the total energy $E=T+V$ is constant. Calculate dE/dt, set it equal to zero, and show that the resulting equation reduces to Eq. (1).