# DIFFERENTIAL EQUATIONS HOMEWORK 7
# ADDITIONAL PROBLEMS

2006 SPRING

(1) In this problem we will consider the linear independence of solutions to the linear (homogeneous, constant coefficient) system of (first-order) differential equations

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

(a) Suppose that $A$ is a $2 \times 2$ matrix, and that $\lambda_1$ and $\lambda_2$ are its eigenvalues. ($\lambda_1$ and $\lambda_2$ may or may not be distinct.) Let $\mathbf{v}_1$ and $\mathbf{v}_2$ (respectively) be corresponding eigenvectors. Let

$$\mathbf{x}_1 = e^{\lambda_1 t}\mathbf{v}_1,$$

and

$$\mathbf{x}_2 = e^{\lambda_2 t}\mathbf{v}_2.$$

Show that $W[\mathbf{x}_1, \mathbf{x}_2](t)$ is non-zero for all $t$ if and only if $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a linearly independent set.

(b) If $A$ is an $3 \times 3$ matrix, suppose that $\lambda_1, \ldots, \lambda_3$ are its eigenvalues and $\mathbf{v}_1, \ldots, \mathbf{v}_3$ (respectively) be corresponding eigenvectors. Let

$$\mathbf{x}_i = e^{\lambda_i t}\mathbf{v}_i,$$

for $i = 1, \ldots, 3$. Show that $W[\mathbf{x}_1, \ldots, \mathbf{x}_3](t)$ is non-zero for all $t$ if and only if $\{\mathbf{v}_1, \ldots, \mathbf{v}_3\}$ is a linearly independent set.