21-260 **Differential Equations** **D. Handron**

Review #2 Spring 2003

Closed book and notes; calculators not permitted.
Be sure to show all work and explain your reasoning as clearly as possible.

1. **(30 points)** Consider the first order differential equation

$$\frac{dy}{dt} = \frac{t}{2y+1}$$

   (a) Find the general solution.

   (b) Find the particular solution with $y(1) = \frac{3}{2}$.

   (c) Is this the unique solution with $y(1) = \frac{3}{2}$? How can you tell?

2. (a) Solve the initial value problem

$$xy' + 3y = x^3, \qquad y(1) = 10$$

   (b) What can you say about the existence and uniqueness of a solution to the above differential equation with initial condition $y(3) = 2$? What about $y(0) = 1$?

3. (a) Use isoclines to draw the slope field for the differential equation

$$y' = \frac{1}{4}x^2 + y^2 - 1.$$

   Sketch the solution curve passing through the point $(0, 0)$.

   (b) Sketch the phase line for the differential equation $y' = (y-1)(y+1)(y+2)^2$ Label each critical point as stable, semi-stable or unstable.

4. Consider the differential equation

$$\frac{dy}{dx} = (y-1)(y+2)^2(y^2+1).$$

   (a) Draw the phase line (phase diagram) for this differential equation.

   (b) Let $y(t)$ be the solution satisfying the initial condition $y(0) = 0$. Can the value of $y(t)$ ever be less than -2? Why or why not?

5. Find the general solution for the differential equation

$$\frac{dy}{dx} = \frac{x(y^2+1)}{2y}$$

6. Consider the second order, linear differential equation

$$y'' - 2y' - 8y = 0.$$

Find the general solution, and the particular solution satisfying $y(0) = 0$ and $y'(0) = 0$.

7. Consider the differential equation

$$\frac{dy}{dt} = y + f(t),$$

where $f(t)$ is a positive, increasing function.

(a) We can find an approximate solution with initial condition $y(1) = 1$ using either Euler's Method or the Improved Euler's Method (a.k.a. the second order Runge-Kutta method).

Let $t_0 = 1$ and $y_0 = \tilde{y}_0 = 1$. Let $t_1 = t_0 + \Delta t$. Let y_1 be the Euler's Method approximation to $y(t_1)$ and $\tilde{y}_1$ the Improved Euler's Method approximation.

Which is greater y_1 or $\tilde{y}_1$? How did you determine the order?

(b) This is a linear differential equation. If $f(t) = e^t$, find the exact solution satisfying the initial condition $y(1) = 1$.