21-241 **Matrix Algebra** **D. Handron**

Exam #2 Review
February 11, 2004 **Name:**_____

Closed book and notes; calculators not permitted.

1. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}.$$

Let $A = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$.

(a) Find a basis for $\text{Col}(A)$ and thus determine the dimension of the image. (Hint: $\text{Col}(A)$ is spanned by $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$.)

(b) Find $\text{Nul}(A)$.

2. (a) Find all solutions to the equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 & 5 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 2 & 0 & 4 & 6 & 1 \\ 1 & 2 & 0 & 7 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$$

(b) Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$. What is the kernel of A? (Write your answer as a span of vectors if you can.)

3. $T : \mathbb{R}^2 \to \mathbb{R}^2$ is the linear transformation such that

$$T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 7 \end{bmatrix}$$

and

$$T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

(a) What is the matrix for this linear transformation? (Hint: $\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.)

(b) Is the linear transformation T invertible? If so, what is the inverse transformation?

4. A, B and C are 2×2 matrices such that

$$ABC = \begin{bmatrix} 5 & -8 \\ 7 & -11 \end{bmatrix} \quad \text{and} \quad (BC)^{-1} = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

What are the matrices A and C?

5. Consider the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \\ -8 & 12 \end{bmatrix}$$

(a) Find a basis for $\text{Col}(A)$.

(b) Find a basis for $\text{Nul}(A^T)$.

6. In each part below, there is a set of vectors $\mathcal{B}$, and a subspace W. In each case, determine if $\mathcal{B}$ is a basis for W. If it is, determine the dimension of the subspace. Otherwise, indicate why the set is not a basis.

(a) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$. $W = \mathbb{R}^3$.

(b) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. $W = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right)$.

(c) $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 16 \end{bmatrix} \right\}$. $W = \text{span}\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 16 \end{bmatrix} \right)$.

7. Let

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -5 & 4 \\ 3 & -2 & 1 & -2 \end{bmatrix}.$$

(a) Find a basis for $\text{Nul}(A)$.

(b) Find a basis for $\text{Col}(A)$.

(c) What is the dimension of $\text{Nul}(A)$? $\text{Col}(A)$?
What is $\dim(\text{Nul}(A)) + \dim(\text{Col}(A))$? Why?

8. $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ are bases for $\mathbb{R}^2$.

(a) If $\mathbf{x} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, find $[\mathbf{x}]_{\mathcal{B}}$ and $[\mathbf{x}]_{\mathcal{C}}$.