MATRIX ALGEBRA FINAL EXAM REVIEW
ADDITIONAL PROBLEMS — HINTS

2005 SPRING

(1) (a)

(b) A the orthogonal projection onto the subspace P has two eigenvalues: 0 and 1. Why? Vectors in P are mapped to themselves by the projection, so they are all eigenvectors with eigenvalue 1. Vectors orthogonal to P are mapped to the zero vector, so they are eigenvectors with eigenvalue 0.
The way to complete this problem is to diagonalize the standard matrix for the transformation. (I'll call it A.) Normally, when we diagonalize a matrix, we are given the matrix to start with, and we factor the given matrix into a product of three matrices: $A = PDP^{-1}$.
The way we do that is by computing the eigenvalues and eigenvectors of A. The columns of the matrix P are a basis of eigenvectors for A, and D is a diagonal matrix whose diagonal entries are the eigenvalues of A. (The order corresponds to the order of the eigenvectors in P.)
In this case we don't know the matrix A, but by thinking geometrically about the projection, we figured out the eigenvalues (0 and 1). We also know the eigenspaces (P and $P^{\perp}$).
You can find a basis for W and a basis for $W^{\perp}$. Joioning these basis togeter will give a basis for $\mathbb{R}^3$, consisting of eigenvectors of A (even though we still don't know what A is yet!).
Using these eigenvectors and eigenvalues, construct a diagonalization of A. Then compute the two products. The resulting matrix is A.
For some examples, see your lecture notes from April 28 and April 30.

(2) This problem is very similar to 1(b). First let me note a correction. The problem meant to ask for the *reflection* across the given subspace (I'll call it W).

A reflection is a linear transformation with two eigenvalues, +1 and -1. As with a projection, vectors in the subspace W are unaffected by the reflection, they are eigenvectors with eigenvalue +1.

Unlike a projection, vectors orthogonal to W are eigenvectors with eigenvalue -1 (rather than 0).

Proceeding as above, find a basis for W and a basis for $W^{\perp}$. Combine them to form a basis for $\mathbb{R}^4$. Use this basis of eigenvectors, and the corresponding eigenvalues to construct a diagonalization of the standard matrix. Multiply out to find out what the standard matrix is.

(3)