MATRIX ALGEBRA HOMEWORK 3
ADDITIONAL PROBLEMS

2005 SPRING

(1) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, defined by $T(\mathbf{x}) = A\mathbf{x}$, vor some $m \times n$ matrix A. Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ be a set of vectors such that $T(\mathbf{v}_i) = \mathbf{b}_i$ for $i = 1, \ldots, p$.

Show that if

$$V = \begin{bmatrix} \mathbf{v}_1 & \ldots & \mathbf{v}_p \end{bmatrix}$$

and

$$B = \begin{bmatrix} \mathbf{b}_1 & \ldots & \mathbf{b}_p \end{bmatrix},$$

then $AV = B$.

(2) Suppose that $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a *linearly independent* set of vectors such that $T(\mathbf{v}_i) = \mathbf{b}_i$ for $i = 1, \ldots, p$.

Show that the standard matrix A for the linear transformation T can be determined from this information. [Hint: The columns of a matrix are linearly independent if and only if its reduced echelon form has a pivot in each column. What $n \times n$ matrices (in recuded echelon form) have a pivot in each column?]

(3) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}.$$

Find the standard matrix for T.