MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#8

GSI: CHRISTOPHER EUR, DATE: 10/21/2016

STUDENT NAME: 42

Problem 1. (5 points) Let $\mathcal{P}_2$ be as usual. Consider the bases $B = (1+x, x^2-x, x^2+x)$ and $C = (1-x, 1+x, x^2+2x)$. Find the change of basis matrix $\underset{C\leftarrow B}{P}$ (which converts coordinates w/r/t B into coordinates w/r/t/ C). [Hint: the diagram below may make your life easier, where $E = (1, x, x^2)$ is another basis of $\mathcal{P}_2$ that is easy to work with]

$$\begin{tikzcd}
\mathcal{P}_2 \arrow[r, "\mathrm{Id}"] & \mathcal{P}_2 \arrow[r, "\mathrm{Id}"] & \mathcal{P}_2 \\
\mathbb{R}^3 \arrow[u, "B", "\sim"'] \arrow[r, "(\underset{E\leftarrow B}{P})"'] & \mathbb{R}^3 \arrow[u, "E", "\sim"'] \arrow[r, "(\underset{C\leftarrow E}{P})"'] & \mathbb{R}^3 \arrow[u, "C", "\sim"']
\end{tikzcd}$$

Problem 2. (5 points) Find all eigenvalues and corresponding eigenvectors of the matrix $\begin{bmatrix} 4 & 2 \\ -5 & -3 \end{bmatrix}$.

#1. We have $\left(\underset{E\leftarrow B}{P}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $\left(\underset{E\leftarrow C}{P}\right) = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

We want $\left(\underset{C\leftarrow B}{P}\right) \underset{\substack{\uparrow \\ \text{from the diagram}}}{=} \left(\underset{C\leftarrow E}{P}\right)\left(\underset{E\leftarrow B}{P}\right) = \left(\underset{E\leftarrow C}{P}\right)^{-1}\left(\underset{E\leftarrow B}{P}\right)$. So, row reduce the right side of:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1 & 1 & -1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 2 & -1 & 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & -1/2 & 1/2 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & -3/2 & -1/2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 3/2 & 1/2 & \\ 1 & -3/2 & -1/2 & I \\ 0 & 1 & 1 & \end{array}\right]$$

$$\Rightarrow \boxed{\begin{bmatrix} 0 & 3/2 & 1/2 \\ 1 & -3/2 & -1/2 \\ 0 & 1 & 1 \end{bmatrix}}$$

#2. $\det\begin{bmatrix} 4-\lambda & 2 \\ -5 & -3-\lambda \end{bmatrix} = (4-\lambda)(-3-\lambda) + 10 = \lambda^2 - \lambda - 12 + 10 = \lambda^2 - \lambda - 2$
$= (\lambda-2)(\lambda+1)$

Eigenvalues: $\boxed{\lambda = 2, \quad \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$ ← corresp. eigenvect.

$\boxed{\lambda = -1, \quad \begin{bmatrix} 2 \\ -5 \end{bmatrix}}$

$\ker\begin{bmatrix} 2 & 2 \\ -5 & -5 \end{bmatrix} \to \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\ker\begin{bmatrix} 5 & 2 \\ -5 & -2 \end{bmatrix} \to \begin{bmatrix} 2 \\ -5 \end{bmatrix}$