MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#1

GSI: CHRISTOPHER EUR, DATE: 2/5/2019

STUDENT NAME: It's Cold!

Problem 1. (5 points) Let $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 1 \\ 4 & 1 & 1 \end{bmatrix}$. Is $\vec{u}$ in the subset of $\mathbb{R}^3$ spanned A?

Problem 2. (5 points) If true, prove it; if false give a counterexample: If $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ are in $\mathbb{R}^4$ and $\vec{v}_3$ is not a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_4$, then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ is linearly independent.

(1) Soln 1: By inspection: $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$.

Hence $\vec{u}$ in the span of col. of A.

Soln 2: $\left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & -3 & 5 & -2 \\ 0 & -1 & 3 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & 1 & -1 & 2 \\ 0 & -1 & 3 & -2 \\ 0 & 0 & -4 & 4 \end{array}\right]$

no pivot in aug. $\Rightarrow$ consistent $\Rightarrow$ $A\vec{x} = \vec{u}$ has soln

$\Downarrow$

$\vec{u} \in$ span of col. of A.

(2) False:

Consider: $\{v_1, v_2, v_3, v_4\} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

Not lin. indep. since the list has a zero vec.

but v_3 still not lin. comb. of $\{v_1, v_2, v_4\}$

$\boxtimes$