

Coupling Smiles

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The first part of the talk will be concerned with the link between implied volatilities and the volatility of the underlying asset. Such a link is of practical interest since it relates the fundamental quantity for pricing derivatives (the spot volatility) which is not observable, to directly observable quantities (the implied volatilities). From a mathematical point of view, it relates information about the law of a positive martingale (the implied volatilities), to the representation of its sample paths as an Itô integral (the spot volatility).

In a second part, we look at an application of this result. As a motivating example, consider the world three major currencies, EUR, JPY, and USD, and their three corresponding exchange rates. An elementary arbitrage argument gives any of the three exchange rates as a function of the other two. We are interested in the similar problem for options on these currencies. More precisely, we would like to reconstruct the implied volatility smile of one currency given the other two.

Parts of the talk are based on a joint work with Nicole El Karoui and on discussions with Andres Villaquiran.