A Synopsis for the Talk: "Extracting the Zeros under a Law of Motion by Geometric Programming"

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1 Outline of the Talk

1. Text Examples & Motivations
2. A Dynamical System under Uncertainty with Perturbations
3. Building a GP Model for the Zeros
4. Numerical Results from GovPX Treasuries Data
5. Zero Curves from *EpiSolutions, Inc*

2 A Motivation for Geometric Progamming Modeling

The ordinary *bootstrap* method for computing forward rates from zero rates generates *posynomial* equations as introduced in an area of optimization termed *geometric programming* invented by Duffin, Peterson, and Zener [6].

$$\text{posynomial disc. fns } e^{-z_k(t_k-t_0)} = \prod_{i=0}^{k-1} x_{i,i+1}^{(t_{i+1}-t_i)},\ k=1,\dots \text{ express the forward rates}$$
$$z_k(t_k-t_0) = \sum_{i=0}^{k-1} f_{i,i+1}(t_{i+1}-t_i),\ \text{where } x_{i,i+1}=e^{-f_{i,i+1}} \text{ in Tables 2–4.} \tag{1}$$

Note that the are n equations in m unknowns ($n = m$ =5). Ordinary bootstrapping does not work when $n \neq m$, eg., if there were no 0.5 time T–Bill.

Table 1: Hull Example Data

Time in Yrs	Annual Coupon	Market Price
0.25	0	97.5
0.50	0	94.9
1.00	0	90.0
1.50	8	96.0
2.00	12	101.6

Table 2: Continuous Discounting Expressions for Zero Rates z_i

t_1	t_2	t_3	t_4	t_5	Price
0.25	0.50	1.00	1.50	2.00	↓
$e^{-.25z_1}$					0.975
	$e^{-.5z_2}$				0.949
		e^{-z_3}			0.900
	$.04e^{-.5z_2}$	$+.04e^{-z_3}$	$+1.04e^{-1.5z_4}$		0.960
	$.06e^{-.5z_2}$	$+.06e^{-z_3}$	$+.06e^{-1.5z_4}$	$+1.06e^{-2z_5}$	1.016
0.10127	0.10469	0.10536	0.10681	0.10808	← Zeros

Table 3: Posynomial Equations from Continuous Discounting: $x_i = e^{z_i}$

t_1	t_2	t_3	t_4	t_5	Price
0.25	0.50	1.00	1.50	2.00	
$x_1^{-.25}$					0.975
	$x_2^{-.5}$				0.949
		x_3^{-1}			0.900
	$0.04x_2^{-.5}$	$+0.04x_3^{-1}$	$+1.04x_4^{-1.5}$		0.960
	$0.06x_2^{-.5}$	$+0.06x_3^{-1}$	$+0.06x_4^{-1.5}$	$+1.06x_5^{-2}$	1.016

2.1 Some Methods for Computing the Zeros

Filipović [8] provides a table based upon a report from the Bank for International Settlements listing zero curve extraction methods of 12 Central Banks, [1].

The state–of–the–art of the spline approach appears in Delbaen and Lorimier [3] and

Table 4: Forward Rates and Zero Rates

t_0	t_1	t_2	t_3	t_4	t_5
0	0.25	0.50	1.00	1.50	2.00
Change	0.25	0.25	0.50	0.50	0.50
zeros	0.10127	0.10469	0.10536	0.10681	0.10808
forwards	f_{01}	f_{12}	f_{23}	f_{34}	f_{45}
forwards	0.10127	0.10811	0.10603	0.10971	0.11189

Table 5: Forward Rate Curve Fitting Procedures of 12 Central Banks

Central Bank	Curve Fitting Procedure
Belgium	Nelson–Siegel, Svensson
Canada	Svensson
Finland	Nelson–Siegel
France	Nelson–Siegel, Svensson
Germany	Nelson–Siegel, Svensson
Italy	Nelson–Siegel
Japan	Smoothing Splines
Norway	Svensson
Spain	Nelson–Siegel(before 1995), Svensson
Sweden	Svensson
UK	Svensson
USA	Smoothing Splines

Lorimier [16]. The Nelson–Siegel and Svensson functions are the following ones.

Nelson–Siegel Forward Rate Function

$$FR(\beta_0, \beta_1, \beta_2, \tau_1) = \beta_0 + \beta_1 \, e^{-t/\tau_1} + \beta_2 \, (t/\tau_1) \, e^{-t/\tau_1}$$

Svensson Forward Rate Function

$$FR(\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2) = \beta_0 + \beta_1 \, e^{-t/\tau_1} + \beta_2 \, (t/\tau_1) \, e^{-t/\tau_1} + \beta_3 \, (t/\tau_2) \, e^{-t/\tau_2}$$

3 A Linear Differential Equation for the Forward Rate under Uncertainty with Perturbations

First, with no uncertainty of any kind the DE for the price of a bond at time t, $P(t,T)$, paying 1 at time T is:

$$\frac{d}{dt} P(t,T) = r(t)\, P(t,T), \quad P(t,t) = 1, \quad r(\tau) \text{ continuous for } \tau \geq t. \tag{2}$$

All the perfect certainty, zero–bond relationships can be derived from the solution to (2);

$$P(t,T) = e^{-\int_t^T r(s)\,ds}. \tag{3}$$

$P(t,T)$ is also referred to as the *discount function* $DF(t,T)$, while function $r(\cdot)$ is the *instantaneous rate of increase* of the bond price, namely,

$$r(t) = \frac{P_t(t,T)}{P(t,T)}. \tag{4}$$

For convenience just in Table 6 we change notation with current time being 0 and the future time now denoted by t.

Table 6: Relations among the Zeros at time 0, future t.

Discount $DF(t)$	$=$	$P(0,t)$
Spot Rate $SR(t))$	$=$	$-(ln\, DF(t))/t$
$DF(t))$	$=$	$e^{-t\, SR(t)}$
Forward Rate $FR(t))$	$=$	$-\frac{d}{dt}\, ln\, DF(t) = -\frac{DF'(t)}{DF(t)}$
$DF(t))$	$=$	$e^{-\int_0^t FR(s)ds}$
$SR(t)$	$=$	$\frac{1}{t} \int_0^t FR(s)\, ds$
$FR(t)$	$=$	$t\, \frac{\partial SR(t)}{\partial t} + SR(t)$

We now return to the $\{t,\ T\}$ time regime. We consider a T-differential model for nonstochastic uncertainty under defining data perturbations within a class of unknown admissible parameters. It is analogous to a class of SDE's where differentiation is with respect to the T variable, not the current time t. Table 7 presents some contrasts to the way each approach treats various features.

<u>Nonstochastic Uncertainty with Perturbations</u>

$$\tilde{f}(s,\omega|t) = f(t,s),\ \ s \geq t \tag{5}$$

$$\frac{d\tilde{f}}{dT}(T|t) = \alpha + \beta \tilde{f}(T|t) + w(T),\ \ T \geq 0,\ \ \tilde{f}(0,0,) = r(0) = r_0,\ \ t = 0,$$

where $w(T)$ is an unknown function of the perturbations acting on the model, and the coefficients, α, β and spot rate r_0 satisfy the following constraints.

$$\begin{aligned} &0 < \alpha_* \leq \alpha \leq \alpha^*,\ \ \beta_* \leq \beta \leq \beta^* < 0, \\ &0 < r_* \leq r_0 \leq r^*. \end{aligned} \tag{6}$$

For illustration we consider here the class of *impulse functions*, where we denote the number of corresponding perturbation variables by L with the associated breakpoints given by:

$$\{t_0, t_1, ..., t_L\}\,,\ \text{where}\ t_{i-1} < t_i,\ \ i = \overline{1, L};\ \ t_0 = 0,\ \text{the current time.} \tag{7}$$

Define the piecewise constant perturbation function, $w(T)$ by:

$$w(T) = w_i,\ \ w_* \leq w_i \leq w^*,\ \text{for all}\ T \in [t_{i-1},\ t_i],\ \ i = \overline{1, L}, \tag{8}$$

where w_*, w^* are preassigned bounds for the perturbations.

Substituting (8) into (5) and applying the Cauchy formula gives the following expression for the forward rate function.

$$\begin{aligned} &\tilde{f}(T,\omega|t) = r_0 e^{\beta T} + \tfrac{\alpha}{\beta}\left(e^{\beta T} - 1\right) + \\ &+ \tfrac{e^{\beta T}}{\beta} \sum_{j=1}^{i-1} \left(e^{-\beta t_{j-1}} - e^{-\beta t_j}\right) w_j + w_i \tfrac{e^{\beta(T - t_{i-1})} - 1}{\beta},\ \ T \in [t_{i-1}, t_i]\,,\ \ i = \overline{1, L}. \end{aligned} \tag{9}$$

The parameter vector ω to be solved for in an optimization is the following list.

$$\left\{r_0, \alpha, \beta;\ \ w_i,\ \ i = \overline{1, L}\right\},\ \text{with}\ \beta\ \text{usually fixed in advance.} \tag{10}$$

3.1 Yield Expressions with Forward Rate Perturbations Variables

As Table 6 portays the spot rate function $R(t,T)$ with $t=0$ has the following form.

$$\begin{gathered}
R(0,T) = \frac{1}{T}\int_0^T f(t,s)ds = \\
\frac{1}{T}\left(\frac{e^{\beta T}-1}{\beta} r_0 + \left(\frac{e^{\beta T}-1}{\beta^2} - \frac{T}{\beta}\right)\alpha + \right. \\
\left.\sum_{k=1}^{i-1}\left(\frac{e^{\beta(T-t_{k-1})}-e^{\beta(T-t_k)}}{\beta^2} - \frac{t_k - t_{k-1}}{\beta}\right) w_k + \left(\frac{e^{\beta(T-t_{i-1})}-1}{\beta^2} - \frac{T-t_{i-1}}{\beta}\right) w_i\right) \\
T \in [t_{i-1}, t_i],\ i = \overline{1,L},
\end{gathered} \tag{11}$$

3.2 A Price–Based Bond Data Optimization Problem

Assume that the observed data are the prices of the bonds, notes, and bills, which we shall refer to in general simply as *bonds*. Assume that there are N number of bonds and that for bond i let $nc(i)$ denote the number of coupon payments occurring at times $\left\{t_{ij} :\ j = \overline{1, nc(i)}\right\}$. Usually $T_i = t_{i,nc(i)}$.Let,

$$P_i = P(T_i | t, c_i, q_i), \quad T_i > t, \quad i = \overline{1,N}. \tag{12}$$

denote the price of instrument i having time to maturity, T_i, coupon rate c_i and frequency of coupon payments q_i, which is typically 0.5 years.

For bond i having coupon rate c_i with frequency of coupon payments q_i its current price is therefore given by:

$$P_i = 100P(0,T_i) + c_i q_i \sum_{j=1}^{nc(i)} P(0,t_{ij}), \quad T_i > t, \quad i = \overline{1,N}. \tag{13}$$

When expressed in term of the forward rate function, the modeled current price of bond i with respect to the forward rate function taking $t=0$.

$$P_i(T_i) = 100e^{-\int_0^{T_i} f(t,s)ds} + c_i q_i \sum_{j=1}^{nc(i)} e^{-\int_0^{t_{ij}} f(t,s)ds}, \quad i = \overline{1,N}. \tag{14}$$

The following definition places the given coupon and maturity dates within the appropriate perturbation interval.

Definition 3.1 *Given* $\{t_{ij}\}_{j=1}^{nc(i)}, [T_i]$, $i = \overline{1,N}$, *set* $arg(t_{ij})\,[arg(T_i)] = m$, *where* $t_{ij}\,[T_i]$ *is contained in the unique perturbation interval* $[t_{m-1}, t_m]$. *Should* $t_{ij}\,[T_i] = t_m$, *then set* $arg(t_{ij})\,[arg(T_i)] = m$, *recognizing that* $t_{ij}\,[T_i] \in [t_{m-1}, t_m]$.

The following notation for the known coefficients in simplifies the mathematical expressions.

$$
\begin{aligned}
&-a_r(T,\beta) = \tfrac{e^{\beta\,T}-1}{\beta} \text{ and } -a_\alpha(T,\beta) = \tfrac{e^{\beta\,T}-1}{\beta^2} - \tfrac{T}{\beta} \\
&-a_k(T,\beta) = \tfrac{e^{\beta\,(T-t_{k-1})}-e^{\beta\,(T-t_k)}}{\beta^2} - \tfrac{t_k - t_{k-1}}{\beta},\ k = \overline{1, j-1} \\
&-b_i(t,\beta) = \tfrac{e^{\beta\,(T-t_{i-1})}-1}{\beta^2} - \tfrac{T-t_{i-1}}{\beta},\ i = \overline{1,N}, \text{ so we can write,}
\end{aligned}
\tag{15}
$$

Using these expressions (14) becomes

$$
\begin{aligned}
&P_i(T_i,\omega) = 100\times \\
&\exp\left(a_r(T_i,\beta)\,r_0 + a_\alpha(T_i,\beta(t))\alpha + \right. \\
&\left.\sum_{k=1}^{arg(T_i)-1} a_k(T_i,\beta)w_k + b_{arg(T_i)}(T_i,\beta)w_{arg(T_i)}\right) \\
&\quad + c_i q_i \sum_{j=1}^{nc(i)} \exp\Big(a_r(t_{ij},\beta)\,r_0 + a_\alpha(t_{ij},\beta)\,\alpha + \sum_{k=1}^{arg(t_{ij})-1} a_k(t_{ij},\beta)w_k \\
&\qquad + b_{arg(t_{ij})}(t_{ij},\beta)w_{arg(t_{ij})}\Big), \quad i = \overline{1,N}.
\end{aligned}
\tag{16}
$$

Remark 3.1 *As is well known, for example [11, Section 4.9], observed prices must include accrued interest, ie, one must use the so–called "dirty prices".*

4 Making a Connection to Geometric Programming Constructs

Let us first review the basic construction of a geometric programming model.

4.1 Posynomial Geometric Programming

Posynomial geometric programming GP is recognized as a very broad class of optimization problems which is useful in many applications, see for example [6, 10].

Table 7: Contrasts: SDE–derived Properties to a Dynamical System under uncertainty with perturbations

Item	Stochastic	Dynamic
type of uncertainty	stochastic process	unknown function from a class of perturbations
model for spot rate	SDE	DE with nonstochastic uncertainty & perturbations
norm of uncertainty	Expectation	minimax or other norm other norm
nonarbitrage condition	risk free measure	constraints in an extremal problem
moments of uncertainty	drift & volatility	minimax amplitude of perturbations
other features	computing non–negative forward rates not always possible	construct additional constraints
Consistency: Börk and Filipović, [9]	attainable	open question

The classical *posynomial* primal GP problem is the following one.

$$\begin{aligned} (GP) \quad \text{minimize} \quad & g_0(t) \\ \text{subject to} \quad & g_k(t) \leq 1, \quad k = 1, 2, ..., p \\ & t_i > 0, \qquad i = 1, 2, ..., m \end{aligned} \tag{17}$$

where

$$g_0(t) = \sum_{i=1}^{n_0} c_i t_1^{a_{i1}} ... t_m^{a_{im}} \tag{18}$$

$$g_k(t) = \sum_{i=n_{k-1}+1}^{n_k} c_i t_1^{a_{i1}} ... t_m^{a_{im}}, \quad k = 1, 2, ..., p. \tag{19}$$

The exponents a_{ij} are arbitrary real constants, and the coefficients c_i are positive. GP has a formal dual program having linear constraints and an objective function which is concave under the logarithmic transformation. The interior point method [14] developed to solve both primal and dual GP's has been implemented and applied in this study. GP itself is transformably convex under the change of variables $t_j = exp(z_j)$, and convex programming solution methods also apply.

4.2 Introducing Geometric Programming Structure into the Dynamical Term Structure Model

The primal GP variables of which there are $L+2$ shall be denoted by $\{x_i\}$. The specific definition of these variables is as follows.

$$x_1 = e^r, x_2 = e^{\alpha}, \text{ and } x_{2+i} = e^{w_i}, \quad i = \overline{1, L}. \tag{20}$$

The number of geometric programming variables depends only on the spot rate variable, the α variable, and the number of points in the partition of the planning horizon. This number is independent of the number of bonds present.

Using this definition $P_i(T_i, \omega)$ in (16) becomes:

$$P_i(T_i) = 100 \times x_1^{a_r(T_i,\beta)} x_2^{a_\alpha(T_i,\beta)} \left(\Pi_{k=1}^{arg(T_i,\beta)-1} x_{k+2}^{a_k(T_i,\beta)}\right) x_{arg(T_i,\beta)+2}^{b_{arg(T_i)}(T_i,\beta)} +$$

$$c_i q_i \sum_{j=1}^{nc(i)} x_1^{a_r(t_{ij},\beta)} x_2^{a_\alpha(t_{ij},\beta)} \Pi_{k=1}^{arg(t_{ij},\beta)-1} x_{k+2}^{a_k(t_{ij},\beta)} x_{arg(t_{ij},\beta)+2}^{b_{arg(t_{i,j})}(t_{ij},\beta)} = \tag{21}$$

$\widehat{u}_i + \sum_{j=1}^{nc(i)} \widehat{u}_{i,j}$, where the definition of the $\widehat{u}$ terms is clear.

Note that terms in (21) are posynomials, noting that each coefficient, such as c_i is positive.

For convenience of primal GP constraints and related to (21), we define posynomials $\left\{u_i,\ u_{ij} : i = \overline{1, N}, j = \overline{1, nc(i)}\right\}$ according to the following definition:

$$P_i(T_i)\ \hat{P}_i^{-1}(T_i) = u_i + \sum_{j=1}^{nc(i)} u_{i,j}, ie., u_i = \hat{u}_i\ \hat{P}^{-1}(T_i) \text{ and } u_{ij} = \hat{u}_{ij}\ \hat{P}^{-1}(T_i). \tag{22}$$

Remark 4.1 *It is conjectured that yield curve models of Nelson–Siegel type, together with their extensions are amenable to equivalent geometric programming formulations for determining the yield curve, provided that one or more of the parameters are fixed. For example, in [4] the authors found grounds for fixing the exponentail decay parameter λ_t at the value 0.002.*

It is very likely that a GP model will arise if the two decay parameters $\tau_i,\ i = 1, 2$ in the Extended Nelson–Siegel–Svensson $ENSS$ employed by Bolder and Stréliski [2, EQ13].

5 Structure of the GP Model

For purposes of this synopsis only a connection has been made to the constructs necessary for development of a GP–optimization model. The the geometric programming model will be presented during the talk and any request for a hard copy will be fulfilled.

Here are some of its features.

- The objective function is based upon the shortest observed yield to maturity, a mean reversion ratio based upon the longest observed yield to maturity, and a measure of smoothness placed upon the sought–for perturbations.

- There are one–sided inequality constraints relating computed prices to observed prices. The direction of the inequalities is given in (17).

- Necessarily there are one–sided inequality constraints whose direction is reversed from (17). In the GP literature these type of constraints are referred to as *reversed constraints*, and they lead to nonconvexities. In general these are difficult to handle, but here the structure of coupon payments and principal at termination suggests an efficient heuristic based on arithmetic–geometric mean inequality approximations.

The latter two sets of inequalites are referred to as *arbitrage constraints* in related optimization approaches.

A valuable reference on the interest rate conventions for many countries is the Krgin book [15]. The widespread, world–wide use of the Nelson–Siegel, Svensson, and spline methods is summarized in [8, TABLE 1]; see also [7]. The topic of forecasting the term structure is addressed in Diebold and Li's paper [4]. Another approach to forecasting in a dynamical systems model under uncertainty with perturbations is discussed in [13, 14.10 CHAPTER NOTES].

Many computations have been performed, but they will be summarized and reported later, particularly focusing on comparative studies with results appearing in [3],[16], [2], [17],and [12].

Some of the numerical implementations of the geometric programming models developed in this paper have a sparse dual–problem matrix having 1,174 rows, 4,804 columns, 504,733 nonzeros, and degree of difficulty of 3650, see Duffin, Petersen, Zener [6] and Duffin, [5].

6 Geometric Programming Computation on Hypothetical Bond Data

A simple example along with an extracted spot rate curve is presented in Tables 8, 9, and 10. It is merely meant to illustrate a particularly simple calculation with rather incomplete data. While the accuracy is acceptable, not enough bonds have been included to obtain a less volatile spot rate curve. A more complete bond market for this case has developed in the ensuing months.

In addition Japanese Treasury data for the Trading Date 20021113 were obtained from the web site of the Japan Securities Dealers Association($JSDA$ at www.jsda.org). The data contained only five-year, ten-year and 20-year Japanese Government Bonds whose liquidity is not questioned and whose total number is 164.

In our numerical experiments with the Japanese data we followed the *actual/actual* day count convention (analogous to Wall Street Journal data) from which bond "dirty prices" were calculated. For checking purposes we calculated the Internal Rate of Returns (IRR) and found agreement with all those appearing in the $JSDA$ data. (With respect to Wall Street Journal data there were a small number of bonds for which there was not approximate agreement with the published $IRR's$).

Analogous to Nelson-Siegel and results of other methods we compute a forward rate function which is used to obtain "computed bond prices" (as well as the spot rate curve). Comparing the 164 computed bond prices for the Japanese Government Bonds to calculated "dirty prices" (from observed clean prices) gave a Mean Absolute Percentage Error ($MAPE$) of 0.0100. Here *yield to maturity* shall simply mean the zero spot rate.

Table 8: Hypothetical Bond Data with Computed Bond Present Values

Bond	Bond Price	Bond Maturity Date(yrs)	Annual Coupon	Coupon Number	Coupon Date	Computed Present Values
1	98.85	0.08				98.8500
2	96.25	0.25				96.2500
3	104.95	3.54	16.5	1	0.041	104.9420
				2	0.541	
				3	1.041	
				4	1.541	
				5	2.041	
				6	2.541	
				7	3.041	
				8	3.541	
4	101.81	4.211	16.5	1	0.211	101.8043
				2	0.711	
				3	1.211	
				4	1.711	
				5	2.211	
				6	2.711	
				7	3.211	
				8	3.711	
				9	4.211	

Table 9: Features of the Computed Solution: Listing the components of the Present Value(PV) of principal & coupon parts: Comparing with Observed Prices

Bond 3	Maturity Date	3.5400	Price PV is 54.804181
Coupon 1	Date Paid	0.0410	Coupon PV is 8.198429
Coupon 2	Date Paid	0.5410	Coupon PV is 7.717684
Coupon 3	Date Paid	1.0410	Coupon PV is 7.064156
Coupon 4	Date Paid	1.5410	Coupon PV is 6.437797
Coupon 5	Date Paid	2.0410	Coupon PV is 5.879267
Coupon 6	Date Paid	2.5410	Coupon PV is 5.379349
Coupon 7	Date Paid	3.0410	Coupon PV is 4.934747
Coupon 8	Date Paid	3.5410	Coupon PV is 4.526479
Computed Full Price Bond 3 is 104.94209 vs OBS 104.95000			
Bond 4	Maturity Date	4.2110	Price PV is 48.985847
Coupon 1	Date Paid	0.2110	Coupon PV is 7.937794
Coupon 2	Date Paid	0.7110	Coupon PV is 7.526568
Coupon 3	Date Paid	1.2110	Coupon PV is 6.878219
Coupon 4	Date Paid	1.7110	Coupon PV is 6.267708
Coupon 5	Date Paid	2.2110	Coupon PV is 5.727960
Coupon 6	Date Paid	2.7110	Coupon PV is 5.234255
Coupon 7	Date Paid	3.2110	Coupon PV is 4.803750
Coupon 8	Date Paid	3.7110	Coupon PV is 4.405024
Coupon 9	Date Paid	4.2110	Coupon PV is 4.037245
Computed Full Price Bond 4 is 101.80437 vs OBS 101.81000			

Table 10: Computed Yield to Maturity at Selected Times

time(yrs)	Yield to Maturity
0.04100	0.15294
0.08000	0.14458
0.21100	0.18283
0.25000	0.15288
0.54100	0.07470
0.71100	0.09928
1.04100	0.10925
1.21100	0.11267
1.54100	0.15398
1.71100	0.14821
2.04100	0.17413
2.21100	0.16798
2.54100	0.16488
2.71100	0.16359
3.04100	0.16673
3.21100	0.16522
3.54100	0.16988
3.71100	0.16874
4.21100	0.16947

7 Forms of GovPX Data During June 03

Typical forms of the Treasury Data from the GovPX source are presented in the next two subsections.

7.1 Bills Data During June 03

Table 11: CUSIP 912795NT4 Bill Maturity 10/23/2003

PART1	00013	Record Number
PART2	912795NT4	Cusip
PART3	10/23/2003	Maturity
PART4	0.000	Coupon
PART5	T3MONR	* Product Type, Alias, Active Code, Settlement
PART6	08/05/2003	Last Trade Date
PART7	15:52:20	Last Trade Time
PART8	T	H-Hit or T-Take
PART9	0.9250	Last Trade Price
PART10	0	Aggregate Volume
PART11	0.9300	Bid Price
PART12	0.9200	Ask Price
PART13	0.9250	Mid Price
PART14	0.942	Mid Yield
PART15	- 0.0050	Price Change same time yesterday
PART16	- 5	Yld Chg BP are 1/10ths of a bp same time yesterday
PART17	0.9575	High Price
PART18	0.8975	Low Price
PART19	0.9250	Tokyo Open Price

*T-N-B: TBill TNote TBond Aias: 10Y identifies security trading sector
Active Code: N-Off-the-Run, A-Active, W-When Issued
Settlement: C-Cash,N-Next Day, W-When-Issued, R-Next Day

7.2 Notes and Bonds Data During June 03

Table 12: CUSIP 912810FE3 Bond Maturity 08/15/2028

PART1	0166	Record Number
PART2	912810FE3	Cusip
PART3	08/15/2028	Maturity
PART4	5.500	Coupon
PART5	B30YN	* Product Type, Alias, Active Code, Settlement
PART6	10/25/2001	Last Trade Date
PART7	00:00:00	Last Trade Time
PART8	T	H-Hit or T-Take
PART9	101.48437500	Last Trade Price
PART10	0	Aggregate Volume
PART11	99.79687500	Bid Price
PART12	99.85937500	Ask Price
PART13	99.82812500	Mid Price
PART14	5.513	Mid Yield
PART15	- 1.32812500	Price Change same time yesterday
PART16	+ 98	Yld Chg BP are 1/10ths of a bp same time yesterday
PART17	101.64062500	High Price
PART18	99.48437500	Low Price
PART19	101.42187500	Tokyo Open Price

*T-N-B: TBill TNote TBond Aias: 10Y identifies security trading sector
Active Code: N-Off-the-Run, A-Active, W-When Issued
Settlement: C-Cash,N-Next Day, W-When-Issued, R-Next Day

7.3 Computed Zero Curve Current Time Monday 061003

In this recent run there were a total of 137 T–Bills, Notes, and Bonds having deleted the callable issues.

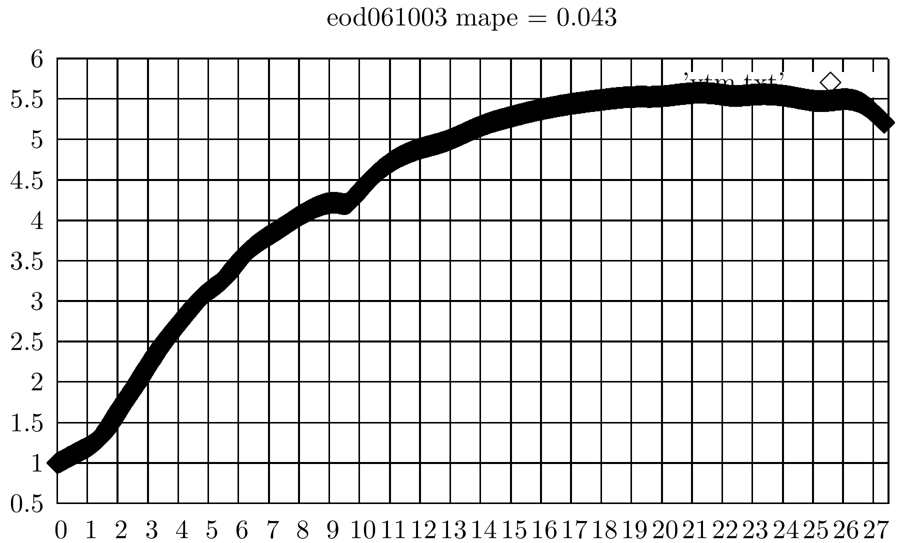

Figure 1: Zero Curve October 6,2003 GovPX Data 137 Bills, Notes & Bonds

Remark 7.1 *Given the maturity date under semi–annual coupon payments as appearing in Table 12, we can establish all the required coupon payment dates. The very last one just before the settlement date (called the current time) is used for the dirty price calculation, and all those after the current time are used in the bond's present value calculation.*

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