Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH I 21-292

Homework 1: Due Wednesday January 21.
Formulate the following as linear programs: Optional: use Gurobi to solve the first problem.

Q1 A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B.

At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours.

The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Unsold production is placed in stock. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

Formulate the problem of deciding how much of each product to make in the current week as a linear program. Then

Solution: let x, y be the number of units of products X, Y produced, respectively.

The amount in stock at the end of the week is

$$30 + 90 + x + y - 75 - 95 = x + y - 50.$$

The amount of processing needed on machine A is

$$50x + 24y.$$

The amount of processing needed on machine B is

$$30x + 33y.$$

So the problem can be formulated

$$\begin{aligned} \text{Maximise } & x + y. \\ \text{Subject to } & 50x + 24y \leq 2400. \\ & 30x + 33y \leq 2100. \\ & x \geq 75 - 30 = 45,\ y \geq 95 - 90 = 5. \end{aligned}$$

Q2 A contractor is working on a project, work on which is expected to last for a period of T weeks. It is estimated that during the jth week, the contractor will need u_j hours of labor, $j = 1, \ldots, T$, for this project. The contractor can fulfill these requirements either by hiring laborers over the entire T week horizon (called steady labor) or by hiring laborers on a weekly basis each week (called casual labor) or by employing a combination of both. One hour of steady labor costs c_1 dollars; the cost is the same each week. However, the cost of casual labor may vary from week to week, and it is expected to be $c_{2,j}$ dollars/hour, during week $j = 1 \ldots, T$. Formulate the problem of fulfilling the labor requirements at minimum cost as a linear program.

Solution: let x be the number of hours per week of steady labor purchase and let y_j be the number of hours per week of casual labor purchased in week j. The total cost of this is

$$c_1 T x + \sum_{j=1}^{T} c_{2,j} y_j.$$

During the week there will be $x + y_j$ hours available. So the problem is

$$\begin{aligned} \text{Minimise } & c_1 T x + \sum_{j=1}^{T} c_{2,j} y_j. \\ \text{Subject to } & x + y_j \geq u_j, j = 1, 2, \ldots, T. \\ & x, y_j \geq 0,\ j = 1, 2, \ldots, T. \end{aligned}$$